On the Effective Description of Large Volume Compactifications

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Abstract: We study the reliability of the Two-Step moduli stabilization in the type-IIB Large Volume Scenarios with matter and gauge interactions. The general analysis is based on a family of $\mathcal{N} = 1$ Supergravity models with a factorizable Kähler invariant function, where the decoupling between two sets of fields without a mass hierarchy is easily understood. For the Large Volume Scenario particular analyses are performed for explicit models, one of such developed for the first time here, finding that the simplified version, where the Dilaton and Complex structure moduli are regarded as frozen by a previous stabilization, is a reliable supersymmetric description whenever the neglected fields stand at their leading F-flatness conditions and be neutral. The terms missed by the simplified approach are either suppressed by powers of the Calabi-Yau volume, or are higher order operators in the matter fields, and then irrelevant for the moduli stabilization procedure. Although the power of the volume suppressing such corrections depends on the particular model, up to the mass level it is independent of the modular weight for the matter fields. This at least for the models studied here but we give arguments to expect the same in general. These claims are checked through numerical examples.

We discuss how the factorizable models present a context where despite the lack of a hierarchy with the supersymmetry breaking scale, the effective theory still has a supersymmetric description. This can be understood from the fact that it is possible to find vanishing solution for the auxiliary components of the fields being integrated out, independently of the remaining dynamics.

Our results settle down the question on the reliability of the way the Dilaton and Complex structure are treated in type-IIB compactifications with large compact manifold volumina.

Keywords: Supergravity Models, Supersymmetry Breaking, Superstring Vacua, dS vacua in string theory

Contents

1   Introduction

The great progress that string theory phenomenology has achieved in recent years is in contrast with the several approximations these studies have always implicit, being one of the most drastic to neglect a large subset of fields. The first candidates to be neglected are the string excitations and Kaluza-Klein (KK) modes which are always very heavy fields, with masses dictated by the string and compactification scales. Then, decoupling [1] and symmetry arguments can be used to argue for an "effective" theory, although these have not been properly integrated out [2]. At this level the truncation is fine since these fields and their couplings are easy to be identified. Nevertheless, the Supergravity (SUGRA) theory describing the remaining light modes is still very complicated such that an explicit and precise study continues to be extremely difficult. A further truncation is then compulsory but now over fields whose masses and couplings are highly sensitive to the point we stand in the field space, so that this procedure starts to be less clear.

This is precisely the philosophy adopted in the stabilization of moduli fields in string compactifications. In this context the moduli to be fixed by tree level effects in the superpotential, flux induced terms [3] which are quantized and therefore naturally of order

one in Planck units, are regarded as completely frozen. Then the rest of the fields are stabilized using non-perturbative effects, that are naturally suppressed. As presented this Two-Step procedure seems alright due to the hierarchy between the sources of the dynamics stabilizing each sector; however, the moduli couplings and even their masses are not as easy to spot as in the case of the string and KK modes, then a more careful study is required. In recent years several studies has tackled this issue from different perspectives (see for example [4–16]). Particularly in [10, 12] things were settled down for models where the perturbative part of the superpotential takes a tiny value at the vacuum. There it was pointed out that this tuning on the superpotential, firstly motivated in the seminal work of Kachru-Kallosh-Linde-Trivedi (KKLT) [17] as such to obtain good phenomenological Vacuum Expectation Values (VEV) for the moduli, is also a necessary requirement for the reliability of the Two-Step procedure, as it ensures a mass hierarchy between the two sets of fields. These results apply to the several sequels of KKLT where the VEV of the superpotential is quite small, and the fields to be fixed by the fluxes are completely disregarded focusing only on the second step where the remaining fields are stabilized.

An important point here is the fact that the theory used in the second step is still a $\mathcal{N} = 1$ SUGRA theory, so the first step is supposed to proceed in a supersymmetric (SUSY) way. From this point of view the tuning on the superpotential can be understood using the fact that the SUSY breaking scale in these KKLT scenarios is dictated mainly by this VEV, so that being small the heavy fields decouple from SUSY breaking effects and the effective theory is approximately SUSY [13].

Following this logic, things seems more subtle for models where the neglected fields acquire masses that are of the same order of the masses of the fields to be stabilized and the SUSY breaking scale. In this class of models fall the Large Volume Scenarios (LVS) [18] where the masses and scales are mainly ruled by the size of the compact manifold developing exponential sized volume.

The LVS has become one of the paradigms of string moduli stabilization being widely studied (see for instance [19–23] and more recent progress in [24–29]). The main appealing feature of this scenario is the fact that it is not necessary to claim for a tuned VEV for the superpotential in order to obtain a low SUSY breaking scale. However, as usual in any moduli stabilization scenario in type-IIB orientifold compactification, the Dilaton and Complex structure moduli are regarded as frozen although their masses lie on the same scale off the remaining field masses, as can be understood from the SUGRA contributions to the masses, coming like $m^2 \sim \langle e^K |W|^2 \rangle$, with K the Kähler potential and W the superpotential. Still, from the structure of the scalar potential, in the large volume limit, it is possible to argue for a decoupling [18] and systematic treatments on this issue have been worked out [8, 10]. The idea in this cases is the realization of a weak coupling between the two sectors so that the stabilization of one side be almost insensitive to the other one.

In absence of gauge interactions such a decoupling is achieved if the system with two sectors, identified by H, the "heavy" fields, and L, the remaining ones in the effective theory, is described by a Kähler invariant function, $G = K + M_P^2 \log \left(|W|^2 / M_P^6 \right)$, M_P the reduced

Planck mass, with a factorizable form, i.e.,

$$G(H, \bar{H}, L, \bar{L}) = G_H(H, \bar{H}) + G_L(L, \bar{L}). \qquad (1.1)$$

This scenario was proposed by Binetruy et al. [30] and extensively studied by Achucarro et al. in several works [6, 8, 9, 14, 16]. In [8, 10] it was pointed out that a Two-Step procedure in the LVS scenario, for the pure moduli case, can be validated through the same generic arguments since all mixing terms in G between the Kähler sector and the Dilaton and/or the Complex structure moduli are suppressed either by powers of the volume or by non-perturbative effects. Despite few comments in [9] these studies where restricted to the case where no gauge interactions nor matter fields were involved. Remains, then, and is the aim of this paper, a systematic study for these more realistic scenarios.

We will follow the same philosophy of [10, 12] by performing a proper integration of the H fields and compare the resulting theory with the simple naive theory were the H fields are simply frozen. Recently Brizi et al. studied the way fields can be integrated out in a SUSY fashion for SUGRA theories [13]. One of their results is that as far as there is a hierarchy between the mass of the fields integrated out and the SUSY breaking scale, one can neglect these breaking effects and integrate the fields in a SUSY way, so that the effective theory, at first approximation in a derivative expansion, is still a SUGRA theory. They, moreover, found that the integration of chiral multiplets proceeds, again at first order, through a chiral equation of motion (e.o.m.) which coincides with the usual flat expression, $\partial_H W = 0$. In our situation this cannot be the case as the neglected terms come like W, which we suppose naturally large, an therefore no longer negligible. However, if the Two-Step procedure is approximately right, the effective theory once the H fields are integrated out should be also approximately SUSY. Then, despite the lack of a chiral e.o.m. that can be exploited in order to make the procedure fully SUSY manifest, as was done in [12], we follow a manifestly SUSY procedure by keeping the auxiliary fields in the Lagrangian and integrating out simultaneously all components of the H chiral multiplets. This not only leads to a straightforward identification of the effective theory as an approximate SUGRA theory but also turns out to simplify the integration of the H multiplets, at least in a schematic form, in the case where gauge interactions are involved. The fact that for the factorizable models the effective theory continues to be a SUGRA one, at least approximately, then can be understood from the fact that the weak coupling between the two sectors allows to have approximately vanishing solutions for the H auxiliary fields, independently of the remaining dynamics. These models present, therefore, an exception to the general case studied in [13] where is the hierarchy between the scales what suppresses the SUSY breaking effects from the H sector, but we leave a fully superfield understanding of this situation for a forthcoming paper [31].

The first thing to be noticed, even before turning on the gauge interactions, is that the presence of matter fields clashes with the factorizability of the system as their wave functions in general depend on fields from both sectors and a large mixing can be realized. However, if the wave function somehow turns out to be suppressed, the mixing between the moduli is still safe, and moreover the dynamics related to the Q multiplets is suppressed such that the independence of the H solutions on the other fields continues to hold. In the

LVS this suppression is indeed realized, with the wave functions coming like $1/\mathcal{V}^n$, with the modular weight n a positive number and $\mathcal{V}$ the volume of the compact manifold [32]. In general one can expect, then, that the corrections to the simplified model be modular weight dependent and the reliability of the Two-Step procedure be constrained by this number. Nicely enough, due to a necessary tuning on the numerical parameters, and the way the matter fields stabilization proceeds, the corrections up to the mass level are not only suppressed but also independent of n!. Since for moduli stabilization issues this order in the fluctuations is enough, this universality on the corrections is an appealing feature of the LVS as a robust playground for moduli stabilization models.

Notice that we are being cavalier using the term effective theory: in case there is a mass hierarchy we could properly speak about an effective theory in the Wilson sense, but in our case things are different and what we call effective theory is simply the one obtained by solving the classical e.o.m. and then plugging back the solution on the original Lagrangian, resulting with what we will call effective Lagrangian. Nevertheless, the approximate factorizability of the Kähler invariant function implies also decoupling of the wave functions, indeed the scalar manifold is at first order factorizable, so that a two derivative approximation is still valid once one requires slow varying solutions in the H sector. This is: the fluctuations in the L sector do not excite, at first order, fluctuations in the H sector.

The introduction of matter fields and gauge interactions in the LVS is far from being just and academic exercise. Indeed, the simplest pure moduli realization of LVS turns out to realize only a deep AdS vacuum, therefore extra ingredients are needed to uplift the vacuum being the more natural ones adding extra fields.[1] Although developing models realizing exponentially sized compact manifold volumina is not as prolific as the ones presenting moduli stabilization *à la* KKLT, mainly due to the rather intricate structure leading the stabilization which makes difficult its generalization, there are a couple of proposals leading to Minkowski vacua, both of them with matter and gauge interactions [21, 29].

The structure of the paper is as follows: section 2 is devoted to generalize the definition of nearly factorizable models with the outlook of introducing matter like fields. With this generalized setup, but without gauge interactions, we show how the decoupling is realized and freezing is a reliable procedure at leading order in a small parameter characterizing the mixing in the Kähler invariant function. The analysis is then particularized to the LVS where, however, there are in principle several unrelated suppression factors for the mixing, namely inverse powers of the volume in the Kähler potential, and non-perturbative effects in the superpotential. Besides of recasting and generalizing some of the results of [8, 10] the study is also performed in a SUSY manifest way that helps to spot easily the SUSY nature of the effective theory and moreover shows itself powerful when dealing with gauge interactions; the main target of the paper lies in section 3 where gauge dynamics are introduced in the study, with the outcome of a constrain on the gauge kinetic function for the general setup in order the Two-Step procedure be reliable. The analysis is then performed for several LVS models where it is shown that the restriction on the gauge kinetic function

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[1]Like in the KKLT scenario [17] it is possible to introduce $\overline{D3}$-branes at the tip of a throat to do the job. This necessarily introduces explicit SUSY breaking terms which one might want to avoid.

is naturally avoided. The section closes with few comments about higher order operators and its reliability in the simplified model. Also few arguments are given in support of the independence of the corrections on the modular weights; section 4 is devoted for the conclusions and some discussion; two appendices are left. One to review the main aspects of the LVS models needed for the study, and to introduce a novel model developing LVS. The second one gives numerical samples showing explicitly the results obtained analytically in the main text.

2   Only chiral multiplets

2.1   Nearly factorizable models

For our purposes it will be convenient to work in a Kähler gauge where the factorizable nature of the $\mathcal{N} = 1$ SUGRA theory be manifested [8, 9, 30]. This leads us to work directly with the generalized Kähler invariant function $G = K + M_P^2 \log\left(\frac{|W|^2}{M_P^6}\right)$, where W is the superpotential, K the Kähler potential and M_P the reduced Planck mass that for simplicity we set to one in the following. The approximate factorizable models are defined as such that only suppressed terms mix two sectors identified by H and L, i.e.,

$$G(H, \bar{H}, L, \bar{L}) = G_H(H, \bar{H}) + G_L(L, \bar{L}) + \epsilon G_{mix}(H, \bar{H}, L, \bar{H}), \qquad (2.1)$$

with ϵ a small parameter. This form was proposed by Binetruy et al. in [30], to describe a SUSY decoupling between the two sectors of fields; however, it lacks from including matter like superfields. Indeed, although the form for G can be justified between moduli sectors, closed string fields, it is not once matter, open string fields, enter in the game since their wave functions depend on both moduli sectors, so factorizability is lost once these fields acquire $\mathcal{O}(1)$ VEV. This situation can be easily taken into account generalizing the factorizable definition of the system with a form for G inspired on the LVS, where the wave function of the matter fields is suppressed by some power of the volume. Thus, in this case, splitting the L sector by the moduli, $\mathcal{M}$, and the matter fields, Q, we propose

$$G(H, \bar{H}, L, \bar{L}) = G_H(H, \bar{H}) + G_{\mathcal{M}}(\mathcal{M}, \bar{\mathcal{M}}) + \epsilon G_{mix}(\phi, \bar{\phi}), \qquad (2.2)$$

with $\phi^I = \{H, \mathcal{M}, Q\}$. This is: the matter fields Q only enter in the suppressed part of G which can depend in all three kind of fields. This form implies that the matter fields enter in the superpotential also in a suppressed way, forbidding $\mathcal{O}(1)$ couplings, e.g., Yukawa like. For the moment let us simply say that fields appearing on such a coupling usually develop vanishing VEV's, which are not dangerous for the factorizability and we postpone its study for the moment. Let us see that a structure like the one defined in eq.(2.2) indeed allows for a decoupling between the two sectors by integrating out the H superfields.

We use the superconformal approach [33–36] to write down the SUGRA Lagrangian since the tensor calculus is similar to the one of rigid supersymmetry. The procedure introduces new degrees of freedom collected in the conformal gravity multiplet with components the graviton, the gravitino and two vector auxiliary fields. Also a compensator chiral multiplet, Φ, is added. By fixing the scalar and spinor components of the compensator and one vector

auxiliary field one recovers the symmetries of ordinary SUGRA. The Lagrangian, then, up to two derivatives can be written as an integration over rigid super-coordinates [34–36],

$$\mathcal{L} = \int d\theta^4 \left(-3e^{-G/3}\Phi\bar{\Phi} \right) + \int d\theta^2 \Phi^3 + h.c. \,. \tag{2.3}$$

The Berenzin integrals, however, are now deformed by extra-terms with dependencies on the components of the gravity multiplet. We will mainly be interested in the scalar component Lagrangian, where these deformations are not relevant, so the calculation is straightforward and easily written like in the global case. With conventions for the metric signature $(+,-,-,-,-)$ and for the chiral fields $\phi = (\phi, \psi, -F^\phi)$ the scalar Lagrangian reads,[2]

$$\begin{aligned}
\mathcal{L} =& G_{M\bar{M}}\partial_\mu\phi^M\partial^\mu\bar{\phi}^{\bar{M}} + G_M F^M \bar{U} + G_{\bar{M}}\overline{F}^{\bar{M}}U \\
&+ \left(G_{M\bar{M}} - \frac{1}{3}G_M G_{\bar{M}} \right) F^M F^{\bar{M}} - 3U\bar{U} - 3e^{\frac{G}{2}}(U + \bar{U}) \,,
\end{aligned} \tag{2.4}$$

where the gauge fixing $\Phi = e^{G/6}$, $F^\Phi = e^{G/6}U$ has been implemented in order to get a canonical normalized Einstein-Hilbert action [34], and we have adopted the convention on the notation for the derivatives on the fields $\partial_M G = G_M$, capital Latin letters running over all fields H, $\mathcal{M}$ and Q. Contrary to the analysis performed in [8, 10] we will keep the auxiliary fields in the game. This not only makes the SUSY nature of the theory to be completely manifest but also will make the following analysis neat, showing more powerful in the case with gauge interactions. Then, to be consistent with SUSY we now integrate out simultaneously both components of the H multiplets, whose e.o.m. are

$$\begin{aligned}
F^i :=& \ G_i\bar{U} + \left(G_{i\bar{N}} - \frac{1}{3}G_i G_{\bar{N}} \right) F^{\bar{N}} = 0 \,, \tag{2.5} \\
H^i :=& \ -G_{i\bar{N}}\partial^2\phi^{\bar{N}} + G_{iM\bar{N}}\partial_\mu\phi^M\partial^\mu\phi^{\bar{N}} + G_{iM}F^M\bar{U} + G_{i\bar{M}}F^{\bar{M}}U \\
& -\frac{3}{2}e^{G/2}(U + \bar{U})G_i + \left[G_{iM\bar{N}} - \frac{1}{3}\left(G_{iM}G_{\bar{N}} + G_M G_{i\bar{N}} \right) \right] F^M F^{\bar{N}} = 0 \,, \tag{2.6}
\end{aligned}$$

with convention of small Latin letters running only over the H fields. The first equation together with the on-shell expression for the compensator auxiliary field, $U = \frac{1}{3}G_M F^M - e^{G/2}$, leads to the on-shell expression for the auxiliary fields, that we write as

$$G_i = e^{-G/2}G_{i\bar{N}}F^{\bar{N}} \,. \tag{2.7}$$

Regarding the approximate factorizability of the function G the e.o.m. for the H fields reads,

$$\begin{aligned}
G_{ij}F^j\bar{U} + G_{i\bar{j}}\overline{F}^{\bar{j}}U + G_{ij\bar{k}}F^j\overline{F}^{\bar{k}} - \frac{1}{3}G_{ij}G_{\bar{N}}F^j\overline{F}^{\bar{N}} - \frac{1}{3}G_M G_{i\bar{j}}F^M\overline{F}^{\bar{j}} \tag{2.8} \\
-\frac{3}{2}e^{G/2}(U + \bar{U})G_i - G_{i\bar{j}}\partial^2\bar{H}^{\bar{j}} + G_{i\bar{j}\bar{k}}\partial^\mu\bar{H}^{\bar{j}}\partial_\mu\bar{H}^{\bar{k}} = \mathcal{O}(\epsilon) \,,
\end{aligned}$$

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[2]For simplicity of notation, here and throughout the paper, we use the same notation for the chiral multiplets and its lowest components, being clear from the context to which one we are referring to.

so that at leading order in ϵ the Q fields completely disappear meanwhile the $\mathcal{M}$ ones only appear in the $G_M F^M$ terms. Here we see that although there is no mass hierarchy between the two sectors the factorizable nature of the G function, and in particular of the Kähler potential, leads to almost decoupled wave functions so that the rotation to normal coordinates does not mix the two sectors. In other words, despite the mass scales of both sectors are the same the kinetic energy from the L sector cannot, at first approximation, excite fluctuations in the H sector (see also [16]), allowing for a two derivative approximation. Looking for slowly varying solutions, using the on-shell expression for the U field and eq.(2.5), one gets

$$G_{ij}F^j\bar{U} + G_{ij\bar{k}}F^j\overline{F}^{\bar{k}} - \frac{1}{2}(U + 3\bar{U})G_{i\bar{j}}\overline{F}^{\bar{j}} - \frac{1}{3}G_{ij}G_{\bar{N}}F^j\overline{F}^{\bar{N}} - \frac{1}{3}G_M G_{i\bar{j}}F^M\overline{F}^{\bar{j}} = \mathcal{O}(\epsilon)\,. \quad (2.9)$$

These, have solutions $F^i = \mathcal{O}(\epsilon)$ implying, from eq.(2.5), that the lowest component of H be solution for

$$\partial_i G = \mathcal{O}(\epsilon)\,, \quad (2.10)$$

so that once we require that W_H, from $G_H = K_H + \log|W_H|^2$, be $\mathcal{O}(1)$ necessarily implies

$$\partial_i G_H = \frac{1}{W_H}(\partial_i W_H + \partial_i K_H W_H) = \mathcal{O}(\epsilon)\,, \quad (2.11)$$

which are nothing but the leading F-flatness conditions one finds by solving the e.o.m. obtained directly from the lowest component potential [10]. Of course, it might happen that these solutions do not exist, or that these do not fix all H fields. We will suppose in the following that this is not the case and all H fields are fixed by (2.11) avoiding issues like the one pointed out in [37]. The solution for the lowest component of H, then, can be cast as $H^i = H_o^i + \epsilon\Delta H^i$, where H_o^i is the solution for $\partial_i G_H = 0$, which is L independent. Plugging back these solutions in (2.4) keeping only up to $\mathcal{O}(\epsilon)$ factors, one gets, with the Greek indices running only over L sector fields,

$$\begin{aligned}
\mathcal{L}_{eff} = &\, G_{\alpha\bar{\beta}}\partial_\mu\phi^\alpha\partial_\mu\bar{\phi}^{\bar{\alpha}} + G_\alpha F^\alpha\bar{U} + G_{\bar{\alpha}}\overline{F}^{\bar{\alpha}}U \\
&+ \left(G_{\alpha\bar{\beta}} - \frac{1}{3}G_\alpha G_{\bar{\beta}}\right)F^\alpha F^{\bar{\beta}} - 3U\bar{U} - 3e^{\frac{G}{2}}(U + \bar{U}) + \mathcal{O}(\epsilon^2)\,, \quad (2.12)
\end{aligned}$$

where we have kept explicitly, in the last term, some $\mathcal{O}(\epsilon^2)$ factors in order not to be forced to split the G function and overload the notation. Notice that the only place where the H^i lowest component solution appear at $\mathcal{O}(\epsilon^0)$ is in the last term. However, expanding around the leading solution, $\partial_i G = \mathcal{O}(\epsilon)$, the corrections will appear only at the $\mathcal{O}(\epsilon^2)$, so we can safely keep only the leading solution for the H^i, i.e., H_o^i, as well in the rest of the Lagrangian being at most $\mathcal{O}(\epsilon)$, and therefore any possible L dependency of the H solution completely disappears.

The effective theory is then described at leading order by the Lagrangian explicitly written in eq.(2.12), that is precisely the one of a SUGRA theory with Kähler invariant function given by the original one but where the H superfields are frozen at their leading solutions,

$$G_{simp}(\mathcal{M}, \bar{\mathcal{M}}, Q, \bar{Q}) = G(H_0, \bar{H}_0, \mathcal{M}, \bar{\mathcal{M}}, Q, \bar{Q})\,, \quad (2.13)$$

with H_0 the constant chiral superfield with vanishing spinor and auxiliary components, and scalar component the leading solution of (2.10). One can further check that the SUSY breaking contributions from the H sector are in fact suppressed compared to the L sector ones. Taking the canonical normalized F-term, $F_c^M = |K_{M\bar{M}}\overline{F}^{\bar{M}}F^M|^{1/2}$ no sum, we have

$$F_c^{\mathcal{M}} \sim \mathcal{O}(1)\,, \quad F_c^Q \sim \mathcal{O}(\epsilon^{1/2})\,, \quad F_c^i \sim \mathcal{O}(\epsilon)\,. \tag{2.14}$$

We are finding explicitly that the resulting theory obtained by integrating out the H fields, regardless the lack of a hierarchy with the $\mathcal{M}$ fields, nor with the SUSY breaking scale [13],[3] can be described at leading order in ϵ by a SUGRA theory whose Lagrangian coincides with one of the simplified version. In particular, in absence of matter fields the simplified version is valid at next to leading order since for the moduli the Lagrangian is of $\mathcal{O}(1)$. In this way we recover the results in [8, 10], with the addendum of making more manifest the SUSY nature of the resulting theory.

We understand, moreover, that it is thanks to the weak coupling between the two sectors that the solutions for the F^i auxiliary fields are suppressed, independently of the remaining dynamics, and we can realize an effective SUSY theory although the masses for the H fields turn our to be of the same order of the SUSY breaking scale.

2.2 Large volume scenario

The $\mathcal{N} = 1$ SUGRA theory obtained in type-IIB orientifold compactifications, where the LVS are realized, is described by a Kähler potential with general structure, including $\mathcal{O}(\alpha'^3)$ corrections [38], given by

$$K = -2\log\left(\mathcal{V} + \xi\,(S + \overline{S})^{3/2}\right) + \mathcal{K}_{CS}\,, \tag{2.15}$$

where $\mathcal{K}_{CS}$ depends only on the Dilaton, S, and complex structure, U^i, but whose explicit form is irrelevant for our study. The Calabi-Yau 3-fold (CY) volume, $\mathcal{V}$, is a Kähler moduli dependent function, such that in case of absence of α' corrections, encoded in the ξ parameter, the SUGRA theory has a no-scale nature.

Although it has been studied some generic properties the theory should satisfied in order to realize vacua with exponentially large volumina [23], explicit realizations of moduli stabilization and characterization of LVS properties are in general based on the so called "Swiss-cheese" manifold compactifications which probably encode all the main features of the LVS models. In the simplest of such manifolds the compact CY is realized as the hypersurface $\mathbf{CP}^4_{[1,1,1,6,9]}$, with $h^{2,1} = 272$ complex structure moduli and $h^{1,1} = 2$ Kähler moduli: one breathing mode, T, and a blow-up mode, t. The volume is in this case [39],

$$\mathcal{V} = \lambda\big(T_r^{3/2} - t_r^{3/2}\big)\,, \tag{2.16}$$

where $\lambda = 1/9\sqrt{2}$, but we leave it in this form in order to have clearer formulae, and we have introduced the notation $\phi_r = (\phi + \bar{\phi})/2$ used here after. In the following we

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[3]Notice that the physical masses, canonically normalized, and SUSY breaking scale, once the L sector breaks it, are all of $\mathcal{O}(1)$.

stick to this form for the volume, mainly to match the existent models but also to have a precise dependency on the L fields being crucial for the analysis. Our results, however, are automatically extended to more complicated Swiss-cheese manifolds since in general with more blow-up modes the volume is generalized as $\mathcal{V} \sim T^{3/2} - \sum_i t_i^{3/2}$. At the qualitatively level our results are also expected to hold for other kind of manifolds.

There is a flux induced tree level superportential given by [40]

$$W_{cs} = \int \Omega_3 \wedge G_3 \,, \tag{2.17}$$

where Ω_3 is the $(3,0)$-form of the compact manifold, and $G_3 = F_3 + iS\,H_3$ the combined three-form-flux. The Kähler moduli, instead, appear in the superpotential only through non-perturbative dynamics, which can also depend on the other moduli. In the spirit of a Two-Step moduli stabilization the Dilaton and Complex structure moduli are regarded as frozen due the superpotential (2.17) with larger dynamics compared to the one encoded in the non-perturbative part. It was found in [10, 12] that in order this procedure be reliable for generic Kähler potential one needs to require a tuning at the vacuum for the tree level superpotential. Indeed, this tuning implies a mass hierarchy which warranties the decoupling between the two sectors. In a more natural scenario the classical superpotential is to get $\mathcal{O}(1)$ VEV, and the decoupling relies in the factorizability of the G function [8, 10]. On the side of the superpotential the mixing between the complex and the Kähler moduli appears only by non-perturbative dynamics, so are naturally suppressed and the approximate factorizability, as defined in eq.(2.1), is safe. On the other hand, the α' corrections of the Kähler potential, breaking the factorizability, are not naturally suppressed. However, whenever we stand at points of the moduli space such that the compact manifold volume is quite large, like in the LVS, these corrections are small an approximate factorizable system is realized. Notice that contrary to the KKLT like models studied in [10, 12], besides the suppression factor coming from the non perturbative dynamics, there is a second one dictated by the size of the volume. This two *a priori* unrelated factors are however connected through the stabilization of the moduli and this relation depends on the particular model, making not viable a generic study as was possible for the KKLT like family of models.

As said before the presence of matter fields breaks the factorizability in the Kähler potential, but in the extended sense given in eq.(2.2) one can still get near factorizability between the H sector, Complex structure and Dilaton moduli, and the L sector, Kähler moduli and charged fields. Indeed the Kähler potential for the matter fields reads [32]

$$K \supset \frac{Z}{\mathcal{V}^n}|Q|^2 \,, \tag{2.18}$$

where Q is a generic charged field, omitting for simplicity possible index contractions, Z is a real function of the Complex structure and the blow-up Kähler moduli t, and the modular weight n is a positive number. Depending on the cycle the $D7$-branes wrap, where the matter is realized, the moduli dependent function Z trivializes or not, and the suppression factor is a power of the volume or just a power of the breathing mode T. For our purposes this schematic form is enough, noticing again that around points in the moduli space where

the size of the volume is quite large the appearance of the Q in the generalized function G is suppressed as far its dependencies in the superpotential be also suppressed. We have, then, in general a third suppressing factor depending on the modular weight.

Let us start by considering the simplest case where W is Q independent, so that the superpotential takes the form, with A a function in general of the H fields,

$$W = W_{sc} + A\,e^{-a\,t}\,, \qquad (2.19)$$

with the non-perturbative part of the superpotential depending only on the *a priori* regarded small modulus. More precisely the breathing mode is supposed extremely large so that such dependencies vanish. If we now suppose that the Kähler moduli stabilization follows like in the pure moduli case, as actually does and even it turns out to happen in realistic models seed by this one (see appendix A), the non perturbative suppression factor coincides with the inverse of the volume and we can define a single suppressing factor given by $\epsilon \sim 1/\mathcal{V} \sim 1/T^{3/2} \sim A\,e^{-a\,t}$, meanwhile $t = \mathcal{O}(1)$. The analog of eq.(2.9) then reads

$$G_{ij}F^i\bar{U} + G_{ij\bar{k}}F^iF^{\bar{k}} - \frac{1}{3}G_{ij}G_{\bar{N}}F^jF^{\bar{N}} - \frac{1}{3}G_{i\bar{j}}G_MF^{\bar{j}}F^M - \frac{1}{2}(U + 3\bar{U})G_{i\bar{j}}F^{\bar{j}}$$
$$\sim (U + F^j + G_NF^N)G_{i\alpha}F^\alpha + G_{i\alpha\bar{\beta}}F^\alpha F^{\bar{\beta}}\,, \qquad (2.20)$$

where as before the capital letters, M and N run over all fields, i, j, over the H ones, and the Greek indexes over the Kähler moduli and the Q fields. In the r.h.s. we have been sloppy with the holomorphic indexes and their contraction being only interested in the leading scaling in ϵ: for example for t, in case Q has a vanishing VEV, we have $G_{t\bar{H}} \sim \epsilon^2$ meanwhile $G_{tH} \sim \epsilon$, so we take the last one for the analysis. The vector and matrix shown in the r.h.s. of eq.(2.20) have the scalings on ϵ,

$$G_{i\alpha} \sim \left(\epsilon^{2/3}(\epsilon + \epsilon^n Q^2), \epsilon + \epsilon^n Q^2, \epsilon^n \bar{Q}\right),$$
$$G_{i\alpha\bar{\beta}} \sim \begin{pmatrix} \epsilon^{4/3}(\epsilon + \epsilon^n Q^2) & \epsilon^{2/3}(\epsilon^2 + \epsilon^n Q^2) & \epsilon^{2/3+n}Q \\ \epsilon^{2/3}(\epsilon^2 + \epsilon^n Q^2) & \epsilon + \epsilon^n|Q|^2 & \epsilon^n Q \\ \epsilon^{2/3+n}Q & \epsilon^n Q & \epsilon^n \end{pmatrix}, \qquad (2.21)$$

i fixed and choosing the ordering $\{T, t, Q\}$. Having in mind that the on-shell solution for the auxiliary fields in the L sector are necessary suppressed by powers of ϵ and looking for approximate SUSY solution in the H sector the leading solution for F^i auxiliary fields has the schematic form

$$F^i \sim G_{i\alpha}F^\alpha \sim \epsilon^{2/3}(\epsilon + \epsilon^n Q^2)F^T + \left(\epsilon + \epsilon^n Q^2\right)F^t + \epsilon^n Q F^Q\,, \qquad (2.22)$$

being again sloppy with the holomorphicity of the indexes, in fact the solution will mix them up in general. This leads us again to conclude that the leading solution for the H lowest component is dictated by the the F^i-flatness conditions,

$$e^{G/2}G_i = G_{i\bar{N}}F^{\bar{N}} \sim \epsilon^{2/3}(\epsilon + \epsilon^n Q^2)F^T + \left(\epsilon + \epsilon^n Q^2\right)F^t + \epsilon^n Q F^Q\,, \qquad (2.23)$$

then, although $e^{G/2} \sim \epsilon$, the on-shell expressions for the F^α show that G_i vanishes at leading order in ϵ. Indeed equation (2.22) tells us about the suppression of the SUSY

breaking contribution from the H sector. As before, we consider that $G_H = \mathcal{K}_{cs} + \log |W_{sc}|^2$ is such that allows for such solutions and fixes the whole set of H fields. Plugging back these in the Lagrangian one has

$$\mathcal{L}_{eff} = G_{\alpha\bar{\beta}}\partial_\mu\phi^\alpha\partial_\mu\bar{\phi}^{\bar{\alpha}} + G_\alpha F^\alpha\bar{U} + G_{\bar{\alpha}}\overline{F}^{\bar{\alpha}}U$$
$$+ \left(G_{\alpha\bar{\beta}} - \frac{1}{3}G_\alpha G_{\bar{\beta}}\right) F^\alpha F^{\bar{\beta}} - 3U\bar{U} - 3e^{\frac{G}{2}}(U + \bar{U}) + \mathcal{O}\big((F^i)^2\big), \quad (2.24)$$

where as before we can keep only the leading solution for the scalar component of H, i.e., $\partial_i G_H = 0$, which are independent of the L fields, being the further corrections negligible. As pointed out before the corrections $\mathcal{O}\big((F^i)^2\big)$ will have not only the SUSY combinations $F\bar{F}$ but also some pure holomorphic and pure non-holomorphic terms that reveal the non-SUSY nature of this effective Lagrangian. We should compare all the $\mathcal{O}\big((F^i)^2\big)$ corrections with terms coming from a SUSY Lagrangian constructed from $G_{sim}(T, \bar{T}, t, \bar{t}, Q, \bar{Q}) = G(H_0, \bar{H}_0, T, \bar{T}, t, \bar{t}, , Q, \bar{Q})$, with H_o a superfield with vanishing spinor and auxiliary field components and whose lowest component is fixed by the leading solution to the F^i-flatness conditions, which are again L independent. This Lagrangian is precisely the one explicitly written in (2.24) with some subleading corrections due to the fact that the correct solution for the scalar component in shifted from the leading SUSY value.

Let us perform explicitly the analysis of the term $|F^t|^2$, so to make the procedure clear, and furthermore spot possible numerical factors necessary to precisely understand the numerical tests performed later on in the appendices. Taking the term $|F^t|^2$ we have that in the simplified version it is lead by

$$G_{t\bar{t}} \sim \frac{3\lambda}{8\sqrt{t_r}\mathcal{V}} + \frac{\partial_t\partial_{\bar{t}}Z}{\mathcal{V}^n}|Q|^2 \sim \epsilon + \epsilon^n|Q|^2. \quad (2.25)$$

This term is also generated in the effective Lagrangian by $|G_{it}F^t|^2$, with leading terms $G_{it} \sim a\, e^{-a\, t}\partial_i A + \frac{\partial_t\partial_i Z}{\mathcal{V}^n}|Q|^2 \sim \epsilon + \epsilon^n|Q|^2$, so there is a further contribution not present in the naive Lagrangian which goes like $(\epsilon + \epsilon^n|Q|^2)^2|F^t|^2$, but is suppressed by $\Delta = \mathcal{O}(\epsilon, \epsilon^n|Q|^2)$. In order to keep track of the numerical factors one has to recall that $A\, e^{-a\, t} = \tilde{A}\lambda\sqrt{t}\frac{\lambda|W_o|}{a}\epsilon$ (see appendix A), with $\tilde{A}$ encoding possible extra factors missed by the analytic procedure, so that in the Q independent parts of the corrections one finds an enhancement. Being precise one finds $\Delta \sim \frac{8}{3}\lambda t^{3/2}\tilde{A}^2\epsilon$. The same analysis is done for all terms, which in the simplified Lagrangian are ruled by the derivatives

$$G_\alpha \sim \left(\epsilon^{2/3}(1 + \epsilon^n|Q|^2), \epsilon + \epsilon^n|Q|^2, \epsilon^n\bar{Q}\right),$$
$$G_{\alpha\bar{\beta}} \sim \begin{pmatrix} \epsilon^{4/3}(1 + \epsilon^n|Q|^2) & \epsilon^{2/3}(\epsilon + \epsilon^n|Q|^2) & \epsilon^{2/3+n}Q \\ \epsilon^{2/3}(\epsilon + \epsilon^n|Q|^2) & \epsilon + \epsilon^n|Q|^2 & \epsilon^nQ \\ \epsilon^{2/3+n}\bar{Q} & \epsilon^n\bar{Q} & \epsilon^n \end{pmatrix}. \quad (2.26)$$

In order to compare terms that are not originally present in the simplified version, e.g., the pure holomorphic $(F^T)^2$ term, we replace one of the auxiliary field to its scaling around the vacuum, which from eq.(2.26) with $F^\alpha \sim e^{G/2}G_{\alpha\bar{\alpha}}^{-1}G_{\bar{\alpha}}$ no sum and (2.22), are

$$F^T \sim \epsilon^{1/3}, \quad F^t \sim \epsilon, \quad F^Q \sim \epsilon Q, \Longrightarrow F^i \sim \epsilon^2 + \epsilon^{n+1}Q^2, \quad (2.27)$$

so that

$$(F^i)^2 \supset \left(\epsilon^2 + \epsilon^{n+1}Q^2\right)\left(\epsilon^{2/3}(\epsilon + \epsilon^n Q^2)F^T + \left(\epsilon + \epsilon^n Q^2\right)F^t + \epsilon^n QF^Q\right), \qquad (2.28)$$

and compare the results with the corresponding one, e.g., F^T, regarding the scaling for the compensator auxiliary field as $U \sim \epsilon$, resulting from its on-shell expression, $U = \frac{1}{3}G_{\bar{M}}F^{\bar{M}} - e^{G/2}$. In this way one finds that all corrections encoded in the $\mathcal{O}((F^i)^2)$ terms are suppressed by $\mathcal{O}(\epsilon, \epsilon^n Q^2)$. As explained in appendix (A.1) in order the stabilization of the moduli indeed proceeds as in the standard case and we can relay on the given scalings, one should impose the condition $\langle Q^2 \rangle \lesssim \epsilon^{1-n}$, so the largest corrections are of $\mathcal{O}(\epsilon)$, independent of the modular weight n!. In this way we generalize the results obtained in [10] working now in presence of matter fields and keeping track of the auxiliary fields. To close this section let us give a look at the canonical normalized auxiliary fields VEV, showing that indeed the SUSY breaking contribution from the H sector is suppressed,

$$F_c^T \sim \epsilon, \quad F_c^t \sim \epsilon^{3/2}, \quad F_c^Q \sim \epsilon^{1+n/2}Q \lesssim \epsilon^{3/2}, \quad F_c^i \sim \epsilon^2, \qquad (2.29)$$

as obtained from eqs.(2.22), (2.26) and (2.27).

3 Chiral and vector multiplets

We now turn on gauge interactions, that can be seen as some gauged isometries of the scalar manifold, which for a $\mathcal{N} = 1$ SUGRA is of the Kähler type. The chiral multiplet transformations are then defined by $\delta\phi^M = \Lambda^A X_A^M$, $\delta\bar{\phi}^{\bar{M}} = \overline{\Lambda}^A \overline{X}_A^{\bar{M}}$, with Λ^A the gauge chiral parameter, X_A^M ($\overline{X}_A^{\bar{M}}$), the holomorphic (antiholomorphic) Killing vectors generating the (gauged) isometry group $\mathcal{G}$, and the gauge group indices running over $A = 1, 2 \ldots \mathrm{adj}(\mathcal{G})$. The associated gauge vector fields transform as $\delta V^A = -i(\Lambda^A - \overline{\Lambda}^A)$, inducing a total transformation on G of the form $\delta G = \Lambda^A X_A^I G_I + \overline{\Lambda}^A \overline{X}_A^{\bar{I}} G_{\bar{I}} - i(\Lambda^A - \overline{\Lambda}^A)G_A = 0$, with G_A denoting derivatives of G with respect to the vector multiplets, and as before the Latin letters running over the chiral multiplets. Gauge invariance of the system then reads[4]

$$G_A = -iX_A^I G_I, \qquad (3.1)$$

a relation that we will use in the following. Notice that if we want to fix the H fields by the conditions $G_i = 0$, all H fields should be necessarily neutral otherwise by gauge invariance, eq.(3.1), the equations turn out to be linearly dependent and leave some unfixed directions.[5] This requirement also avoids the possibility the H fields be sourced by the

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[4]We have not included constant Fayet-Iliopoulos terms ξ_A in eq.(3.1) since they seem not to appear in theories raised from string compactifications.

[5]Notice that one might generalize the way the H fields are fixed by regarding two gauge sectors, say $\mathcal{G} = \mathcal{G}_{\tilde{A}} \otimes \mathcal{G}_A$, so that the H are charged under $\mathcal{G}_{\tilde{A}}$ but neutral under $\mathcal{G}_A$, and the opposite for the L fields. Then fix the H fields by the requirement $G_i = 0$ and $G_{\tilde{A}} = 0$. This is, besides the F-flatness the D-flatness conditions. In this case some of the H field are then stabilized by gauge dynamics and not by the superpotential, as usually regarded for the H sector in string compactification, so we left this situation out of our analysis.

gauge fields in the effective theory. We set, then, in the following $X_A^i = 0$.

Using the conformal formalism the Lagrangian of eq.(2.3) is first of all corrected by the G dependency on the vector multiplets V^A, promoting to covariant derivatives the derivatives in the kinetic terms, i.e.,

$$\partial_\mu \phi^I \to \partial_\mu \phi^I + X_A^I V_\mu^A \,. \tag{3.2}$$

Further terms for (2.3) come from the chiral Lagrangian and from the kinetic part of the vector multiplets Lagrangian. Working in the Wess-Zumino gauge, denoting the vector auxiliary fields by D^A one has the following extra terms to the ones in eq.(2.4) [33]

$$\mathcal{L} \supset G_A D^A + \frac{1}{2} h_{AB} D^A D^B \,, \tag{3.3}$$

with $h_{AB} = Re(f_{AB})$, f_{AB} the field dependent gauge kinetic functions.

3.1 Nearly factorizable models

As before let us first tackle the case of the generic nearly factorizable models where the situation follows as before with the e.o.m. for F^i auxiliary fields not changed, eq.(2.5), and for the scalar components the analogous of eq.(2.8) now reads,

$$-G_{i\bar{j}}\partial^2 \overline{H}^{\bar{j}} + G_{ij\bar{k}}\partial_\mu H^j \partial^\mu \overline{H}^{\bar{k}} + G_{ij}F^j \bar{U} + G_{ij\bar{k}}F^j \overline{F}^{\bar{k}} - \frac{1}{2}(U + 3\bar{U})G_{i\bar{j}}\overline{F}^{\bar{j}}$$
$$-\frac{1}{3}G_{ij}G_{\bar{N}}F^j \overline{F}^{\bar{N}} - \frac{1}{3}G_M G_{i\bar{j}}F^M \overline{F}^{\bar{j}} + \frac{1}{2}D^A D^B \partial_i h_{AB} = \mathcal{O}(\epsilon)\,, \tag{3.4}$$

obtained by using the relation (3.1), the neutrality of the H fields, so that $\partial_i G_A = -iX_A^\alpha G_{\alpha i}$, and regarding that none of the Killing vectors X^α get anomalous large values, i.e., $\mathcal{O}(X_A^\alpha) \sim 1$. We have, then, that the presence of the last terms makes the solution $F^i \sim \epsilon$ no longer automatic; indeed, the e.o.m. tells us that whenever the D-term gets a non-vanishing VEV, which is naturally of $\mathcal{O}(1)$, it back reacts on the H sector, even if neutral, inducing a large F^i-term. In particular the F^i auxiliary fields solutions would depend strongly on the L fields, therefore cannot be simply neglected, as a Two-Step stabilization requires, and have to be properly integrated out. Notice, moreover, that the mixing between the F and the D auxiliary fields makes again explicit the non-SUSY nature of the effective theory, which on the scalar side still is a two derivative description.[6]

The decoupling would be still safe if somehow the dependency of the gauge kinetic function on the H superfields is absent or suppressed by say ϵ. In this case things turn out to be like in the case without gauge dynamics as the solution $F^i \sim \epsilon$ is again valid, and the leading solution for the lowest component is again dictated by the leading F-flatness conditions, that do not depend on the L sector. Thus, the simplified theory, where the H superfields are regarded as frozen at their leading solution, is reliable at leading order, as far as the H superfields are neutral and the frozen value is, up to $\mathcal{O}(\epsilon)$ corrections, the leading solution to the F^i-flatness condition.

Notice that we have not distinguished between the cases with and without breaking of the gauge symmetries. In the study done in [12] it was important to keep track of the gauge

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[6]A similar analysis was done in [13] studying the SUSY description of effective theories.

symmetry breaking as some of the fields in the L sector, regarded as light, acquire heavy masses via the D-term dynamics. Then, one needs to distinguish these fields from the low energy spectrum when speaking about an effective theory integrating these fields out, not being in general possible to freeze them out, together with the H fields. In our case however, modes acquiring masses through this mechanism are not heavier than other fields in the L sector. Indeed, regarding the eigenvalues of h_{AB} of $\mathcal{O}(1)$, their masses should be of the order of the eigenvalues of vector field mass matrix associated to the broken generators,

$$M^2_{AB} = 2G_{I\bar{I}}X^I_A\overline{X}^{\bar{I}}_B\,, \tag{3.5}$$

which are clearly of $\mathcal{O}(1)$. Therefore, there is not need of spotting such fields and no formal distinction appears if the symmetry is broken or not. There might appear a small hierarchy with such fields, like in the case $N_f < N_C$ worked out in section A.2. In this case one can proceed like done in [12] by integrating the massive vector superfields through the superfield vector equation $\partial_V K = 0$ [13] and choosing a convenient gauge fixing, fixing the value of one of the charged fields with a non-vanishing component in the would-be Goldstone direction. Since the Kähler potential has a factorizable form and the H multiplets are neutral the dependency of the solution on the H fields will be necessarily suppressed. Plugging back the solution into the original Kähler potential leads then to a new Kähler potential which however has still a factorizable form, and the same analysis can be performed with only the unbroken symmetries. However, since in our models is the weak coupling between the sectors and not the mass hierarchy what plays the important role in the decoupling, these issues are not relevant for our study, being this more clear if we remember that the effective theory we are speaking about is not a proper one in the Wilson sense.

We find, then, that in order Two-Step procedure be reliable in general[7] for this generic factorizable models, a further restriction is required, namely a suppressed dependency of the gauge kinetic function on the fields to be frozen, something that cannot be justified in general for realistic situations. Nevertheless, it is clear that this general setup as stands so far cannot be connected to a real situation as SUSY is broken at Planck scale. Interestingly enough, in the next section we find that in realistic realizations of nearly factorizable models, namely the LVS, this restriction is avoided naturally.

3.2 Large volume scenarios

In type-IIB orientifold compactifications with $D7$-branes the role of the gauge kinetic functions is played by the Kähler moduli parameterizing the size of the cycles where the branes wrap. The presence of fluxes, however, induces corrections on the gauge kinetic functions that depend on the Dilaton modulus [41], which are supposed to be frozen in the Two-step stabilization spirit. Thus, the condition we found for a reliable decoupling in generic nearly factorizable models is not naturally realized. Since the mixing between the F^i and the D^A auxiliary fields, which is what rises this problem, still appears, if decoupling indeed applies its justification should come from other means. In the previous case, however, we

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[7]We will show in later sections that what really matters is the VEV of the D^A, so decoupling is still save whenever this VEV is tiny.

were forced to strengthen the requirements on the models from the fact that the D^A are naturally of $\mathcal{O}(1)$, as seen by their on-shell expression,

$$D^A = -h^{AB} G_A = i h^{AB} X_A^\alpha G_\alpha \,, \tag{3.6}$$

with the matrix $h^{AB} \equiv h_{AB}^{-1}$, since apart from the matter field contribution, of $\mathcal{O}(\epsilon)$, $D^A \sim G_\alpha = \mathcal{O}(1)$. Even more, this scaling naturally holds at the vacuum, where the following dynamical relation is satisfied [42],

$$\frac{1}{2} \left(M_{AB}^2 + h_{AB} \big(G_{I\bar{J}} F^I F^{\bar{J}} - e^G \big) \right) D^B = G_{AI\bar{J}} F^I F^{\bar{J}} \,, \tag{3.7}$$

everything being naturally of $\mathcal{O}(1)$. On the other hand, for our type-IIB setup all G_α are suppressed by powers of the volume, so in a LVS things seems to be controlled on favor of the decoupling, but even though the decoupling is still not trivially clear. Moreover like in the pure chiral fields case a complete generic analysis is not possible, so we will follow the study separately for explicit models with matter fields realizing moduli stabilization in LVS with vanishing cosmological constant [21, 29]. In this models the main effects from gauge dynamics, besides the non-perturbative superpotential, come from Abelian sectors under which the moduli get charged, so in the following we only consider tree level effects coming from such $U(1)_X$ group. This do not loose the generality of the study as the analysis of other sectors follows in the same way and even more straightforward as the moduli are neutral under those sectors and the dynamics are simpler. As a matter of fact it is common to neglect such non-Abelian sectors by working in a meson field description following D-flat directions.

As usual the symmetry is linearly realized for the matter like fields, instead the moduli, controlling the gauge kinetic functions, can develop a non-linear realization due to a Green-Schwarz (GS) mechanism that cancels pseudoanomalies of the Abelian sectors. In the case of type-IIB superstring with $D7$-branes the Kähler moduli get charged once world volume magnetic fluxes are turned on.[8] The $U(1)_X$ symmetries in the chiral superfields are then described by the holomorphic Killing vectors,

$$X_X^I = i q_X^I \phi^I \text{ no sum} \,, \quad X_X^{T^j} = i \frac{1}{2} \delta_X^{T^j} \,, \tag{3.8}$$

with q_X^I the charge of the chiral superfield ϕ^I and the GS coefficient $\delta_X^{T^j}$ a real parameter associated to the modulus T^j. The gauge invariant Kähler potential for the moduli has then the functional dependency

$$K_{mod} = K \left[T^j + \overline{T}^j + \frac{1}{2} \delta_X^{T^j} V^X \right] \,, \tag{3.9}$$

with V^X the vector superfield associated with the $U(1)_X$ symmetry. To make simpler the analysis we will regard that only one of the Kähler modulus gets charged, but in general is possible that several of them develop non trivial charges. The main effect for moduli stabilization issues is the new contribution induced in the D-term dynamics which enters like a Fayet-Iliopoulos (FI) term but now field dependent [21, 29, 45–49].

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[8]To be precise a topological condition should be realized, namely that the 4-cycle whose volume is characterized by the Kähler modulus has an intersection with the 2-cycle where the world volume flux is non-trivial, in order such a charge be not null [43, 44].

3.2.1 Charged large modulus

The simplest implementations of this kind of models were developed in [21] with the large modulus charged.[9] Although the explicit realization of the models might have several matter fields for our purposes their final out-shot can be cast using only one Q, so that

$$X_X^T = i\frac{1}{2}\delta^T, \quad X_X^t = 0, \quad X_X^Q = q_Q Q. \tag{3.10}$$

By gauge invariance the superpotential does not depend on the Q field so the F-term part of the scalar Lagrangian goes like in the case without gauge interactions, and it is possible to stabilize the Kähler moduli like in the pure moduli case. The scalings shown in eq.(2.26) and eq.(2.27) then hold, and new scalings needed for the analysis are, using eq.(3.1),

$$G_X \sim h_X D^X \sim \delta_T \epsilon^{2/3} + \epsilon^n |Q|^2, \quad G_{Xi} = -iX_X^\alpha K_{\alpha i} \sim \delta_T \epsilon^{5/3} + \epsilon^n |Q|^2. \tag{3.11}$$

Thus a small size for D^X is implied and again is possible to find a suppressed solution for the F^i auxiliary fields, using the on-shell expression for the D^X auxiliary fields,

$$\begin{aligned}
F^i &\sim G_\alpha F^\alpha + \frac{1}{\epsilon}G_X D^X \\
&= \epsilon^{2/3}(\epsilon + \epsilon^n Q^2)F^T + (\epsilon + \epsilon^n Q^2)F^t + \epsilon^n Q F^Q + (\delta_T \epsilon^{-1/3} + \epsilon^{n-1}|Q|^2)D^X, \tag{3.12}
\end{aligned}$$

with the last term $1/\epsilon$ factor coming by regarding the compensator scaling $U \sim \epsilon$. This last term apparently couple strongly the two sector, and might give rise to L dependencies in the H solution; however, it turns out to be inoffensive as explained bellow, so the effective Lagrangian is again given by the simplified version plus corrections of the form

$$\mathcal{O}(F^i)^2 \sim (F_F^i)^2 + \frac{1}{\epsilon}G_X D^X F_F^I + \frac{1}{\epsilon^2}G_X^2 (D^X)^2, \tag{3.13}$$

where by F_F^i we denoted the pure F auxiliary fields contribution, eq.(2.22).

The analysis of the corrections to the simplified version from the $(F_F^i)^2$ part is exactly the same as the one with no gauge interaction since the F-term part of the Lagrangian does not change. Remains, then, only to check the new features coming from the last two terms which enter as corrections to the F^α, D^X and $(D^X)^2$ terms.

Under this setup there are two possible scenarios, depending on the sign of the Q charge, differing by the VEV of Q which is ruled mainly by the minimization of the D-term dynamics, eq.(3.3), which once the D auxiliary fields are integrated out, eq.(3.6), reads

$$V_D = \frac{1}{2\,h_X}(G_X)^2 = \frac{1}{2\,h_X}\left(\frac{1}{2}\delta^T \partial_T K + q_Q \frac{Z}{\mathcal{V}^n}|Q|^2\right)^2, \tag{3.14}$$

where the gauge invariance of the superpotential, $X^I \partial_I W = 0$ is used.

The leading behaviour of the first term in (3.14) is given by $\partial_T K \sim -1/T_r < 0$, and if the charge is of the opposite sign of the GS coefficient the minimization of V_D leads to

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[9]Although the authors of [21] also study models with a charged small modulus, these turn out to lack of realizing a vanishing cosmological constant.

a vanishing VEV for Q. Then, at the minimum the D-term potential scales like $V_D \sim \frac{1}{h_X}\left(\frac{\delta^T}{T_r^2}\right)^2$, and so in order to uplift the $V_F = -\mathcal{O}(\epsilon^3)$ vacuum without sweeping away the moduli stabilization, one requires a tuning $\delta^T \sim \epsilon^{5/6}\sqrt{h_X}$. This, together with the vanishing of $\langle Q \rangle$, implies that the dangerous coupling in (3.12) is in fact suppressed even in the extreme case when $h_X \sim T_r \sim \epsilon^{-2/3}$. From the effective Lagrangian point of view we have that since $G_X D^X/\epsilon \sim \epsilon^2$, independently of the scaling of h_X, and $F_F^i \sim \epsilon^2$ as before, then $G_X D^X F_F^i/\epsilon$ leads to correction to the pure F-term part similar to the ones coming from $(F_F^i)^2$ and which we learned before are suppressed by $\mathcal{O}(\epsilon)$. These also rise corrections to the $G_X D^X$ term of the form $\epsilon G_X D^X$. The term $h_X(D^X)^2$ is corrected by the last term in (3.13) that using (3.11) and the constrain on δ^T is of the form $\epsilon h_X(D^X)^2$. We find, then, that all new possible corrections continue to be suppressed at least by $\mathcal{O}(\epsilon)$. In case the charge turn out to be of the same sign of the GS coefficient the minimization of the D-term potential tends to a cancellation lead by a non-vanishing VEV for Q, which now scales like $\langle Q^2 \rangle \sim \epsilon^{2/3-n}\delta^T$. The uplifting now proceeds from the F-term part with a term scaling like $V_F \supset \epsilon^{2+n}Q^2$, so in order to uplift without spoiling the moduli stabilization a tuning $\delta^T \sim \epsilon^{1/3}$ is required. Due to the Q dependency in the F-term potential the cancellation of the D-term part is not exact, finding in fact that at the vacuum $D^X \sim \epsilon^2$, independently of the scaling of h_X, and therefore $G_X \sim h_X\epsilon^2$. We have then that $G_X D^X \sim h_X\epsilon^4$ and even in the worst case $h_X \sim T \sim \epsilon^{-2/3}$ it is smaller that in the previous case so the correction to the pure F-term part are further suppressed, more precisely are suppressed at least by $\mathcal{O}(\epsilon^{4/3})$, the same happening to the corrections to the $h_X(D^X)^2$ terms. Since the change in F_F^i is negligible, $F_F^i \sim \epsilon^2 + h_X\epsilon^3 \sim \epsilon^2$, the corrections to $G_X D^X$ are still of $\mathcal{O}(\epsilon)$.[10] Interestingly we still find that the corrections are independent of the modular weight and the decoupling is in general safe.

3.2.2 Charged small modulus

Once one allows a charged small modulus things turn out to be more reach and intricate as gauge invariance imply the appearance of matter like fields in the superpotential. Thus the stabilization of the moduli should be revisted, possibly affecting the scalings used above. Again the minimization of the D-term potential, which now takes the form

$$V_D \sim \frac{1}{h_X}\left(-\frac{\delta^t}{2}\partial_t K + q_Q Q\partial_Q K\right) \sim \frac{1}{h_X}\left(-\delta^t\epsilon + q_Q\epsilon^n|Q|^2\right), \qquad (3.15)$$

plays a crucial role in the stabilization of the matter. Notice, however, the change in the sign for the field dependent FI term which makes the implementation of such setup quite different to the usual SUGRA FI model [45–49]. A brief review of models with these characteristics is done in appendix A.

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[10]One might wonder if it is correct to take the scaling of D^X at the vacuum, which for this last case is drastically smaller than the off-shell one. In fact, taking the off-shell scaling (3.11) with the constrain on δ^T one would find corrections of $\mathcal{O}(\epsilon^{2/3})/h_X$ so that in the situation $h_X \sim t \sim 1$ the corrections would be $\mathcal{O}(\epsilon^{2/3})$ not of $\mathcal{O}(\epsilon)$ as claimed above. We have check numerically that indeed this is not the case by taking a sample with $h_X = t_r$, finding still corrections of $\mathcal{O}(\epsilon)$.

$N_F < N_C$ case

In the window $N_F < N_C$, with N_f the number of families and N_C the rank of the gauge group, that we suppose to be an $SU(N_C)$, the system develops an ADS superpotential [50] which in the particular case $N_F = 1$ and $N_C = 2$ takes the following form

$$W = W_{cs} + A\frac{e^{-at}}{\phi^2} + \frac{1}{2}m\rho\phi^2 \,, \tag{3.16}$$

where the chiral matter fields charged under the non-Abelian sector are described by the canonical normalized meson $\phi = \sqrt{2Q\tilde{Q}}$. As before W_{cs} and A are $\mathcal{O}(1)$ functions of H fields. We have also added a mass term, in general H-dependent, which from gauge invariance under the Abelian sector depends on a second matter like field ρ. We normalize the charges such that $\delta^t = 2/a$, thus $q_\phi = -1/2$ and $q_\rho = 1$. This model although proposed in [21] was not studied there, so we take the opportunity to carefully check if indeed leads to a LVS and characterize its vacua (see appendix A). Despite the fact it turns out that as it stands it cannot lead to realistic solutions with vanishing cosmological constant we do the analysis of the decoupling having the model interesting characteristics like the mass term in the superpotential. The modular weights for both matter fields are taken equal and the kinetic gauge function is taken to be $f_X \sim t$. The VEV of ϕ is ruled by the cancellation of the D-term dynamics, meanwhile for ρ a non-vanishing VEV is only possible for a non-zero mass parameter. The moduli are ruled by a potential whose structure is similar to the standard LVS but with different powers of the volume. Nicely enough a fully analytical solution is found in the large volume expansion, with out-shot,

$$e^{-at} \sim \epsilon^{2-n} \,, \quad t \sim 1 \,, \quad \phi \sim \epsilon^{\frac{1-n}{2}} \,, \quad \rho \sim \epsilon^{\frac{5-3n}{6}} \,. \tag{3.17}$$

Contrary to the first two cases, here it is not necessary to argue for a tuning in the GS coefficient, however, the mass term may lead the dynamics and sweep the solution, so should be tuned by $m \sim \epsilon^{\frac{3n-1}{2}}$. With this consideration the following scalings are found,

$$G_\alpha \sim \left(\epsilon^{\frac{2}{3}}, \epsilon, \epsilon^{\frac{1+n}{2}}, \epsilon^{\frac{1+n}{2}}\right), \quad G_{\alpha\bar{\beta}} \sim \begin{pmatrix} \epsilon^{4/3} & \epsilon^{5/3} & \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{3+n}{2}} \\ \epsilon^{5/3} & \epsilon & \epsilon^{\frac{1+n}{2}} & \epsilon^{\frac{5+3n}{6}} \\ \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{1+n}{2}} & \epsilon^n & 0 \\ \epsilon^{\frac{3+n}{2}} & \epsilon^{\frac{5+3n}{6}} & 0 & \epsilon^n \end{pmatrix}, \tag{3.18}$$

with ordering $\{T, t, \phi, \rho\}$, and where for simplicity in the analysis, contrary to the previous examples, we have replaced the matter fields by its VEV to extract the scalings. The mixed derivatives scale as

$$G_{i\alpha} \sim \left(\epsilon^{5/3}, \epsilon, \epsilon^{\frac{1+n}{2}}, \epsilon^{\frac{1+n}{2}}\right), \quad G_{i\alpha\bar{\beta}} \sim \begin{pmatrix} \epsilon^{7/3} & \epsilon^{8/3} & \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{3+n}{2}} \\ \epsilon^{8/3} & \epsilon^2 & \epsilon^{\frac{1+n}{2}} & \epsilon^{\frac{5+3n}{6}} \\ \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{1+n}{2}} & \epsilon^n & 0 \\ \epsilon^{\frac{3+n}{2}} & \epsilon^{\frac{5+3n}{6}} & 0 & \epsilon^n \end{pmatrix}, \tag{3.19}$$

from which we estimate the VEV of the auxiliary fields,

$$F^T \sim \epsilon^{1/3} \,, \quad F^t \sim \epsilon \,, \quad F^\phi \sim \epsilon^{\frac{3-n}{2}} \,, \quad F^\rho \sim \epsilon^{\frac{3-n}{2}} \quad \text{and} \quad F^i \sim \epsilon^2. \tag{3.20}$$

We have also $G_X \sim D^X \sim \epsilon^2$. The canonical normalized auxiliary fields VEV's, encoding the SUSY breaking contribution from each sector, scale as $F_c^T \sim \epsilon$, $F_c^t \sim \epsilon^{3/2}$, $F_c^\phi \sim \epsilon^{3/2}$, $F_c^\rho \sim \epsilon^{3/2}$ and $F_c^i \sim \epsilon^2$. The same analysis followed before shows corrections to the simplified version suppressed by $\mathcal{O}(\epsilon^{2/3})$. Nicely enough, even though we are now dealing with a fourth suppression factor, namely the mass parameter, the leading order for the corrections to the simplified version are again independent of the modular weight. Like in the other cases, this is due to the tuning on the parameters from the requirement of a vanishing cosmological constant, or more weakly, the requirement that the stabilization of the moduli is not spoiled. A related result is the independence on the modular weight of the VEV of non-pertubative superpotential, being in all cases $\mathcal{O}(\epsilon)$, and even the mass term in this last example goes like $m\rho\phi^2 \sim \epsilon^{4/3}$.

The largest corrections we are spotting here come from $G_{i\rho} \sim \phi^2 \partial_i m \sim \epsilon^{\frac{1+n}{2}}$ together with G_{iT} and G_{it} affecting the $G_{T\bar{\rho}}F^T F^{\bar{\rho}} + G_{t\bar{\rho}}F^t F^{\bar{\rho}} + h.c.$ terms in the simplified version. Therefore, in case the mass parameter is absent, or independent of the H fields, the corrections are smaller and turn out to be like in the previous cases $\mathcal{O}(\epsilon)$. In the numerical check done in appendix B, for example, it turns out that the $\mathcal{O}(\epsilon)$ corrections are as important as the ones coming from the mass parameter due to enhancement factors like the ones found before.

$N_C < N_F < 3/2N_F$ case

A realistic model already present in the literature is the Krippendorf-Quevedo model developed in [29], regarding the Seiberg dual description of the theory in the window $N_C < N_F < 3/2N_F$. The model can be minimally described by two matter fields with the following superpotential,

$$W = W_{cs} + Ae^{-a\,t}\rho\phi\,, \tag{3.21}$$

and as before W_{cs} and A can depend on the H fields and are regarded to be $\mathcal{O}(1)$.[11] The charges can be normalized fixing the GS coefficient to be $\delta^t = 2/a$ so that, $q_\phi = -1$ and $q_\rho = 2$. The same modular modular weight is taken for both matter fields and the kinetic gauge function is taken to be $f_X \sim t$. Like in the previous case the VEV of ϕ is ruled by the D-term minimization. A rough analytical approach shows the scalings to be

$$e^{-a\,t} \sim \epsilon^{n+1/2}\,, \quad t \sim 1\,, \quad \phi \sim \epsilon^{(1-n)/2}\,, \quad \rho \sim \epsilon^{(2-n)/2}\,. \tag{3.22}$$

Then for the analysis we have the scalings

$$G_\alpha \sim \left(\epsilon^{2/3}, \epsilon, \epsilon^{(1+n)/2}, \epsilon^{(2+n)/2}\right)\,, \qquad G_{\alpha\bar{\beta}} \sim \begin{pmatrix} \epsilon^{\frac{4}{3}} & \epsilon^{\frac{5}{3}} & \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{10+3n}{6}} \\ \epsilon^{\frac{5}{3}} & \epsilon & \epsilon^{\frac{1+n}{2}} & \epsilon^{\frac{2+n}{2}} \\ \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{1+n}{2}} & \epsilon^n & 0 \\ \epsilon^{\frac{10+3n}{6}} & \epsilon^{\frac{2+n}{2}} & 0 & \epsilon^n \end{pmatrix}\,, \tag{3.23}$$

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[11]See appendix A.2 where is argued that such assumption breaks down whenever the volume turns out to be huge. Here we simply disregard such subtlety which makes obscure the analysis.

with ordering $\{T, t, \phi, \rho\}$ and where like in the previous case we replaced explicitly the VEV of the matter fields so to make simpler the analysis. In the vacuum also $D^X \sim G_X \sim \epsilon^2$, which together with the mixed derivatives with the H sector

$$G_{i\alpha} \sim \left(\epsilon^{\frac{5}{3}}, \epsilon, \epsilon^{\frac{1+n}{2}}, \epsilon^{\frac{2+n}{2}} \right), \qquad G_{i\alpha\bar{\beta}} \sim \begin{pmatrix} \epsilon^{\frac{4}{3}} & \epsilon^{\frac{5}{3}} & \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{10+3n}{6}} \\ \epsilon^{\frac{5}{3}} & \epsilon & \epsilon^{\frac{1+n}{2}} & \epsilon^{\frac{2+n}{2}} \\ \epsilon^{\frac{7+3n}{6}} & \epsilon^{\frac{1+n}{2}} & \epsilon^n & 0 \\ \epsilon^{\frac{10+3n}{6}} & \epsilon^{\frac{2+n}{2}} & 0 & \epsilon^n \end{pmatrix}, \qquad (3.24)$$

imply

$$F^T \sim \epsilon^{1/3}, \quad F^t \sim \epsilon, \quad F^\phi \sim \epsilon^{\frac{3-n}{2}}, \quad F^\rho \sim \epsilon^{\frac{4-n}{2}}, \quad F^i \sim \epsilon^2. \qquad (3.25)$$

The analysis follows like in the previous cases finding corrections suppressed at least by $\mathcal{O}(\epsilon)$, lead by the equivalent ones found in the pure moduli case; indeed, the VEV for the non-perturbative part of the superpotential again scales like $\mathcal{O}(\epsilon)$ and the corrections turn out to be independent of the modular weight. Again the canonical normalized F-terms: $F_c^T \sim \epsilon$, $F_c^t \sim \epsilon^{3/2}$, $F_c^\phi \sim \epsilon^{3/2}$, $F_c^\rho \sim \epsilon^2$ and $F_c^i \sim \epsilon^2$, show the suppressed contribution from the H sector to the breaking of SUSY. Unfortunately the rough analytical approach we followed in finding the solutions is not enough for our computer skills so to allow us to find numerically this kind of vacua and check the conclusions just raised.

Higher order operators and universality in the corrections

So far we have regarded only the lowest order operators in the Kähler potential neglecting contributions of the form $K \supset Q^p/\mathcal{V}^m$ with $p > 2$. Although the introduction of such terms implies to deal with extra suppression factors, as the power m is in principle unrelated to the modular weight n, we can from our results easily spot the effects of integrating out the H fields in such a case. In section 2.2 we found that the simplified version misses terms that are suppressed by $\mathcal{O}(\epsilon^n Q^2)$, where the Q^2 can be seen as fluctuations of the matter fields. This means that the full effective theory realizes terms like $\mathcal{L}_{eff} \supset \epsilon^{2+2n} Q^4$, which affect terms coming from the higher coupling in the Kähler potential with $p = 4$, namely $\mathcal{L}_{simp} \supset \epsilon^{2+m}|Q|^4$, e.g., from $G_{Q\bar{Q}}|F^Q|^2$, telling us that such a term in the Kähler potential is not reliable unless $m < 2n$. Similar comments follow for any other higher couplings. These higher order effective terms can be easily understood by collapsing Feynman diagrams with H fields in the internal lines, finding also the same kind of corrections once higher order operators are regarded in the superpotential; then, a $\mathcal{O}(1)$ Yukawa coupling can induce operators correcting the Q^6 operator in the superpotential and the Q^5 one in the Kähler potential, exactly as is found even in case a mass hierarchy is realized [12].
The above discussion is somehow related to the fact that contrary to the moduli, the derivatives with respect the matter fields always carry a decreasing on the suppression factor controlling the factorization of the theory, since these fields scale with inverse powers of the volume. Then, at some order in the matter fields fluctuations the factorizability is expected to breakdown and the corrections start to be non-suppressed.
Notice that this analysis points out also, and contrary to claim done above, for the possibility the corrections to the lower order operators to be dependent on the modular weights.

Indeed, the corrections of $\mathcal{O}(\epsilon^n Q^2)$ can be seen also as $\mathcal{O}(\epsilon^n Q)\delta Q$ corrections with δQ the fluctuations. Let us see more precisely that in fact up to the mass level, at least for the models studied, the corrections are universal.

For this we write down the scalar potential expanding in fluctuations δQ and δH around the H_o solution,

$$
\begin{aligned}
V &= V_o + V_i \delta H + V_Q \delta Q + V_{Qi} \delta Q \delta H + M_H^2 \delta H \delta H + V_{QQ} \delta Q \delta Q \\
&\sim \epsilon^3 + \epsilon^3 \delta H + \epsilon^{\frac{5+n}{2}} \delta Q + \epsilon^{\frac{5+n}{2}} \delta Q \delta H + \epsilon^2 \delta H \delta H + \epsilon^{2+n} \delta Q \delta Q,
\end{aligned} \tag{3.26}
$$

where we have used the scalings at the vacuum, and the fact that $V_{Qi} \sim V_Q \sim Q V_{QQ}$, valid for the fields not stabilized by the D-term cancellation in which case a larger suppression appear in the V_Q and therefore things are even safer. We also used that the non-canonical mass squared for these matter fields turn out to be always of $\mathcal{O}(\epsilon^{2+n})$ and the largest VEV for the matter is $\mathcal{O}(\epsilon^{\frac{1-n}{2}})$. An expansion in the moduli fluctuation is also implicit in the coefficients but for our present purposes are not relevant. Then an integration at the gaussian level leads to $\delta H \sim \epsilon + \epsilon^{\frac{1+n}{2}} \delta Q$, and plugging these back in the potential we have corrections of the form

$$
\delta V = \epsilon^4 + \epsilon^{\frac{7+n}{2}} \delta Q + \epsilon^{3+n} \delta Q \delta Q, \tag{3.27}
$$

which are suppressed compared to the original operators by $\mathcal{O}(\epsilon)$, independent of n. These as far the largest corrections are concerned, since we took the largest VEV for the Q's. This analysis, in fact, is quite general as uses the largest possible VEV for a generic Q field, in an arbitrary LVS model, therefore the conclusion is expected to hold in a more general context.

4 Conclusions

Our results show how the factorizable models present a framework where freezing of a set of light fields is a reliable procedure. Although several papers have worked on these models [6, 8, 9, 14], we present a generalization which describes the introduction of matter like fields, whose wave function depends in general on fields from both sectors. Then, once these fields enter only in the suppressed part of the generalized Kähler function, eq.(2.2), decoupling between the two sectors is easily understood through the same analysis done in the pure moduli case, with the same conclusion: freezing of the H fields is reliable as far as the H lowest components stand at their leading F-flatness solutions. The corrections, however, contrary to the pure moduli case, where start at the next-to-next-to-leading order in the suppression parameter ϵ, start now at the next-to-leading order. With gauge interactions turned on this conclusion does not change but one has to require the H fields to be neutral and, in case the D-term SUSY breaking be non suppressed, that the gauge kinetic function dependency on these fields be suppressed. These models although not realistic, with a SUSY breaking scale or order the Planck mass, present all the important features needed to understand how the freezing of the Dilaton and Complex structure moduli proceeds in the LVS of type-IIB string compactifications. Nicely enough, in these scenarios the D-term SUSY breaking is naturally suppressed by some powers of the volume

and the constrain found for the gauge kinetic function is avoided.

A somehow unexpected result is that, in all models explicitly studied, the largest corrections to the simplified version, up to the mass level in the matter fluctuations, are independent of the modular weight for the matter fields. This due to the tuning on the parameters and a constrain on the largest possible VEV a Q field can get, both related to the requirement of a vanishing cosmological constant, or the more relaxed and general one of avoiding the destabilization of the vacuum. On the other hand, higher order corrections may depend on the modular weights. These, however, are expected to be irrelevant for moduli stabilization issues and therefore harmless for the reliability of the Two-Step procedure.

Our study for the LVS, although far from being fully generic, shows how the Two-Step stabilization procedure is expected to be safe in realistic and complicated scenarios, and how the corrections are universally independent of the modular weights. It also introduces a simple way to estimate the mistake done by performing quantitative analysis from the simplified version. Still, a more conservative approach can use the fact that in moduli stabilization studies the harder part for the quantitative information is finding the vacua, say numerically. Then, once the vacuum is found through the simplified version one can implement the full model knowing that the solutions are just slightly perturbed. This, in fact, is the procedure followed in the analytical and numerical examples shown in the appendices. With the full information at hand one is safe of missing fine information probably needed for precision tests, for example soft terms ruled by the H fields. Off course, explicit string implementations most likely would need a sort of Two-Step procedure, even if less drastic, due to the huge number of fields. Nevertheless, our study sets down a consistency check for the LVS as a systematic procedure for moduli stabilization.

An interesting result our analysis shows, is the SUSY nature of the effective theory despite the lack of a hierarchy between the energy scales in both sectors, in particular with the SUSY breaking scale. One can understand this from the fact that the main requirement to be satisfied in order the effective theory be approximately SUSY, is that the solution for the auxiliary fields of the multiplets being integrated out be always suppressed compared with the auxiliary fields of the remaining fields, independently of the dynamics of the effective theory. One way of realizing such situation, as found by Brizi et al. [13], is by integrating out fields with large SUSY mass compared to the SUSY breaking scale, driven this last one by the remaining dynamics. In this case is the large inertia from the SUSY preserving sector what shields it from the SUSY breaking effects in the second sector, and the auxiliary field VEV's of these multiplets are suppressed by the masses. In our case, is the weak coupling among the different sectors what ensures that the solution in one side be independent of the other one. Then any back reaction from the SUSY breaking sector is negligible in the SUSY preserving sector. As a matter of fact, in case the mixing parameter ϵ vanishes the solution for the F^i auxiliary fields exactly vanishes and one can even speak about an exact SUSY description of the effective theory, contrary to the case with the hierarchy where there are always corrections to the two derivative SUSY description of the effective theory [13]. In order to put these arguments in a complete SUSY framework, one should understand them from a fully superfield approach, a study left for a forthcoming paper [31].

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A   LVS with matter

This appendix is devoted to resume the principal features of the LVS models with matter and gauge interactions, for details please refer to the original works [21, 29].[12]   In the following we use the more orthodox form for the scalar potential integrating out the full set of auxiliary fields,

$$V = e^G \big(G_{\bar\alpha} G^{\bar\alpha\alpha} G_\alpha - 3 \big) + \frac{1}{2h_X}(G_X)^2 \,, \tag{A.1}$$

with the matrix $G^{\bar\alpha\alpha} \equiv G_{\alpha\bar\alpha}^{-1}$, and the rest of the functions as defined in the main text. We restrict the study to the simplified version of the models, where the Dilaton and Complex structure moduli are regarded as frozen, since our conclusions show that all features we need are encoded here. Moreover we neglect the now constant part $\mathcal{K}_{cs}$ in the Kähler potential, eq.(2.15), changing only by a multiplicative factor the whole F-term scalar potential, and as before we neglect the non-Abelian parts in the D-term dynamics.

The Kähler moduli enter in the superpotential only though non-perturbative effects, exponentially suppressed by the moduli, then any dependency on the large modulus can be neglected. In absence of matter fields the superpotential then looks like

$$W = W_o + A e^{-a\,t} \,, \tag{A.2}$$

with W_o the $\mathcal{O}(1)$ remnants of a flux induced superpotential once the Dilaton and Complex structure are frozen, i.e., $W_o \equiv W_{SC}(U_o^i, S_o)$, and the amplitude A might also depend on these moduli. The standard scenario works without gauge interaction nor matter like fields, then the scalar potential, in a large volume expansion and regarding t and all the parameters as real, reads [18],

$$V_F \approx \frac{8}{3\lambda}\frac{a^2\sqrt{t}|A|^2}{\mathcal{V}}e^{-2a\,t} - 4a\,t\frac{|A\,W_o|}{\mathcal{V}^2}e^{-a\,t} + \frac{3}{2}\frac{\hat\xi|W_o|^2}{\mathcal{V}^3} \,, \tag{A.3}$$

where $\hat\xi = \xi(2S_r)^{3/2}$. The competing effects of these three terms lead to a stable minimum, being crucial that the last term be positive definite, namely $\hat\xi > 0$, which translates to a condition on the kind of CY, in a generic setup, where such minima can exist, i.e.,

as $\xi \sim h^{2,1} - h^{1,1}$ this implies more complex structures than Kähler moduli [23]. The minimization of this potential leads to [18]

$$t \approx \left(\frac{\hat{\xi}}{\lambda}\right)^{2/3} , \quad \mathcal{V} \approx \frac{9\lambda|W_o|}{4a|A|}\sqrt{t}e^{at} , \qquad (A.4)$$

with corrections of order $1/at \ll 1$. The canonical normalized masses $m_T \sim \epsilon^{3/2}$ and $m_t \sim \epsilon$ can be estimated by disregarding the Kähler mixing, i.e., $m_i^2 \sim (K_{ii})^{-1}\partial_i\partial_i V$, using the notation $\mathcal{V}^{-1} \sim \epsilon$ as in the main text. Notice the possibility of realizing low energy SUSY breaking, characterized by the gravitino mass $m_{3/2} = e^{G/2} \sim \epsilon$, with a natural VEV for W driven by the exponentially sized volume. The problem with this setup is the fact that the vacuum turns out to be a deep AdS, of $\mathcal{O}(\epsilon^3)$, as can be seen from the fact that the second term in (A.3) has in the amplitude a factor $at \gg 1$ which enhances it compared to the other two.

A.1 Charged large modulus

Once matter fields are considered, with a Kähler potential like the one in eq.(2.18), even if these turn out not to appear in the superpotential the F-term scalar potential is affected with new terms lead, in a large volume expansion, by

$$V_F \supset e^G G^{Q\bar{Q}}|G_Q|^2 \sim e^G G_{Q\bar{Q}}^{-1}|G_Q|^2 \sim \epsilon^{2+n}|Q|^2 . \qquad (A.5)$$

If such terms are not to sweep off the minimum of the moduli, expected to proceed like in the standard scenario, and potentially realize a zero tuned cosmological constant, these should scale at most as ϵ^3, so we get a constrain on the VEV for the Q's,

$$\langle|Q|^2\rangle \lesssim \epsilon^{1-n} . \qquad (A.6)$$

A superpotential like eq.(A.2) still is possible with a neutral small modulus, and the appearance of the Q fields might be forbidden by gauge invariance, then the generic form for the D-term part of the scalar potential is the one give in eq.(3.14).

Vanishing Q VEV

The simplest and nicely enough also a realistic scenario developing vanishing cosmological constant, is the one where the Q charge has opposite sign of the GS coefficient, so that all terms in the D-term are positive definite, as well the mass term in the F-term potential. Then a vanishing VEV solution for Q is possible.

The uplift, thus, is not possible through (A.5) but from the D-term potential scaling as

$$V_D \sim \frac{1}{h_X}\left(\frac{\delta^T}{T+\bar{T}}\right)^2 \sim \epsilon^{4/3}\frac{\left(\delta^T\right)^2}{h_X} , \qquad (A.7)$$

so that $G_X \sim \delta^T \epsilon^{2/3}$ and $D^X \sim \delta^T \epsilon^{2/3}/h_X$. Then in order this to work as an uplift for the AdS solution of $V_F = -\mathcal{O}(\epsilon^3)$, and no to spoil the moduli stabilization, we find that the GS coefficient should be tuned as $\delta^T \lesssim \epsilon^{5/6}\sqrt{h_X}$, and even in the best case when

$h_X \sim T \sim \epsilon^{-2/3}$ we have a strong constrain $\delta^T \lesssim \epsilon^{1/2}$.

Contrary to the standard scenario no massless modes are realized, as the imaginary part of T is the would-be Goldstone boson eaten up by the now massive vector field of the broken $U(1)_X$. For the Q field the mass is ruled by the term (A.5), so that the canonical mass goes like $m_Q \sim \epsilon$. The gauge breaking scale, encoded in the canonical mass for the vector bosons, is given by $m_X = \sqrt{K_{T\bar{T}} X^T \bar{X}^{\bar{T}} / h_X} \sim \epsilon^{2/3} \delta^T / \sqrt{h_X} \sim \epsilon^{3/2}$.

Non-vanishing Q VEV

When the sign of q_Q is opposite to the one of the GS coefficient the minimization of the potential leads in a good approximation to a cancellation of the D-term potential, driven by a non-vanishing Q VEV, so

$$\langle |Q|^2 \rangle \approx \frac{\delta^T}{q_Q} \frac{\mathcal{V}^n}{Z(t_r) T_r} \sim \epsilon^{2/3-n} \delta^T \,. \tag{A.8}$$

Let us see the structure for the full e.o.m. for Q, so to have a more precise estimation for the VEV of the D-term part of the potential

$$\partial_Q V \approx \partial_Q \left(e^G G_{Q\bar{Q}}^{-1} |G_Q|^2 \right) + \frac{1}{2h_X} G_X G_{QX} \sim \epsilon^{2+n} Q + \frac{1}{h_X} \epsilon^n Q G_X = 0 \,, \tag{A.9}$$

where without loss of generality we regard h_X independent of Q, the discarded term being of order $(G_X)^2$ is any way irrelevant. Then, we find that

$$G_X \sim h_X \epsilon^2 \,, \quad D^X \sim G_X / h_X \sim \epsilon^2 \,. \tag{A.10}$$

We have, then, that the D-term part of the potential scales as $V_D \sim h_X \epsilon^4$ and even in the best case $h_X \sim \epsilon^{-2/3}$ it can not work as uplifting for the moduli vacuum which follow again like in the pure moduli case. This role is now played by the contribution (A.5), scaling as

$$V_{uplift} \sim \epsilon^{2+n} |Q|^2 \sim \epsilon^{8/3} \delta^T \,. \tag{A.11}$$

Again a tuning on the GS coefficient, $\delta^T \lesssim \epsilon^{1/3}$, is required though in this case is milder and furthermore independent of the scaling of h_X.

The physical mass scalings follow exactly like in the previous case, with an important contribution to the mass of Q from the D-term dynamics though, and the canonical normalized mass for the vector boson $m_X \sim \epsilon^{1/2} / \sqrt{h_X}$.

A.2 Charged small modulus

Things are more reach when the small modulus is charged as the the superpotential starts necessarily to depend on the Q fields. As is well known depending on the number of families and the rank of the gauge group the description of the system changes.

$N_f < N_c$ **case**

The following model was already proposed by Cremades et al. in [21], however, it was not discussed there. Here we discuss the stabilization of the fields in detail and try to argue why is not possible to realize Minkowski/dS vacua with this setup.

Working in the window $N_f < N_c$ for a non-Abelian $SU(N_c)$ sector an ADS nonperturbative superpotential [50] is generated, depending on the non-perturbative scale Λ and the chiral fields, Q ($\tilde{Q}$), transforming in the fundamental (antifundamental). It is convenient to work in D-flat directions characterized by the mesonic fields, then choosing the canonical normalized field $\phi = \sqrt{2Q\tilde{Q}}$ [46], and regarding each family as the copy of the others the non-perturbative superpotential takes the form

$$W_{np} = (N_c - N_f) \left(\frac{2\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{\frac{1}{N_c - N_f}} . \tag{A.12}$$

The non-perturbative scale is a function of the moduli which as before we take to be the small modulus. To make easier the following analysis we take the simplest case, namely $N_c = 2$ and $N_f = 1$, then the superpotential takes the general form,

$$W = W_o - A\frac{e^{-a\,t}}{\phi^2} - \frac{1}{2}m\rho\phi^2 , \tag{A.13}$$

where we have included the flux induced superpotential that stabilizes the Dilaton and Complex structure moduli, and encoded some N_c and N_f dependencies in the parameters A and a that are naturally of order one. A possible mass term is also included, which requires a further charged field, ρ, singlet under the $SU(2)$.[13]

Without loss of generality we fix the charge for the modulus by $\delta^t = 2/a$, so the holomorphic Killing vectors are $X_X = i(0, 1/a, -\phi/2, \rho)$. The Kähler potential for the matter fields is taken to be $K \supset \frac{Z(t_r)}{T_r^n}|Q|^2$, so the leading expression for the D-term

$$G_X = -iX^i G_i \approx \frac{1}{2} \left(\frac{1}{a}\frac{3\sqrt{t_r}}{T_r^{3/2}} + \frac{Z(t_r)}{T_r^n}(2\rho\bar{\rho} - \phi\bar{\phi}) \right) , \tag{A.14}$$

where we have also discarded subleading terms in $1/at$.

Despite the mass term for the mesons these cannot be integrated out since the tachionic mass from the D-term dynamics likely dominates. In fact, the minimization of the potential leads to a cancellation of the D-term potential via a non-vanishing VEV for ϕ, analogous to the previous case, but now this happens for the opposite sing of the charge as the sign of the field dependent FI term changes. Then $\langle\phi^2\rangle \approx \frac{3\sqrt{t_r}}{aZ(t_r)T_r^{3/2-n}} \sim \epsilon^{1-2n/3}\phi_o(t_r)^2$, taking *a priori* $\langle\phi\rangle \gg \langle\rho\rangle$ and $\phi_o(t_r)$ encoding factors scaling like $\mathcal{O}(1)$. In order to stress the fact that the dynamics from the mass term are weaker let us discuss first the case $m = 0$ and then see the consequences of turning this term on. Thus, taking the gauge kinetic function

---

[13]The phase between the terms is irrelevant as can be modified by the value of the coefficient and have been fixed only in order to clear up the following formulae meanwhile the coefficients are regarded as positive.

$f_X = t$ the leading D-term potential for ρ once ϕ is fixed is

$$V_D \approx \frac{Z(t_r)^2}{t_r T_r^{2n}} |\rho|^4 .$$

(A.15)

From here on we consider real solutions for the e.o.m. and forget the subscript r, this is later checked numerically for real parameters. Linear terms in ρ would appear only as factors of the mass term with leading expression, coming from the $G^{\phi\bar\phi}|G_\phi|^2$ and $G^{t\bar\phi}G_t G_{\bar\phi} + h.c.$ terms, which once ϕ is fixed reads at leading order

$$-\frac{8}{\lambda^2 Z(t) T^{3/2}} m\,\rho \left(\frac{t\,W_o Z'(t)\phi_o(t)^2}{4T^{3-n}} + 4a\,A e^{-at}\big(\phi_o(t)^{-2} - a\,t Z'(t)\big) \right) .$$

(A.16)

Therefore, for the case we are currently studying a solution $\rho = 0$ is possible. The mass terms for ρ are, at leading order, given by

$$\frac{4m^2\phi_o(t)^2}{\lambda^2 Z(t) T^{9/2-2n}} - \frac{8at A e^{-at}}{9\lambda^2 Z(t)\phi_o(t)^4} \left(\frac{3\sqrt{t}\,W_o\phi_o(t)^2}{Z(t)^2 T^{2n}} - 2a A e^{-at} \right) (Z'(t)^2 - Z(t)Z''(t)),$$

(A.17)

again lead by the G_t and G_ϕ terms. With this fixed solutions for ρ and ϕ we get an effective potential for the moduli, which at leading order in the volume and $1/at \ll 1$ reads,

$$V_{mod} = \frac{8Z(t)^2 a^4 A^2 e^{-2at}}{27\lambda^2 \sqrt{t}\,T^{2n-3/2}} - \frac{2\sqrt{t}\,Z(t)a^2 W_o A e^{-at}}{\lambda^2 T^{3/2+n}} + \frac{3W_o^2 \hat\xi}{2\lambda^3 T^{9/2}} ,$$

(A.18)

where the first two terms come from the $G^{t\bar t}|G_t|^2$ contribution and the last one is the usual α' term from $G^{T\bar T}|G_T|^2$. Interestingly enough the structure of this potential is exactly like the standard LVS, eq.(A.3), with a change in the powers for the large modulus. This change in the powers induces a different scaling for the exponential since the minimization leads to the same scaling of all three terms at the minimum. Indeed we see that at the minimum the exponential will in this case scale like $e^{-at} \sim T^{n-3} \sim \epsilon^{2-2n/3}$, while in the standard scenario $e^{-at} \sim T^{-3/2} \sim \epsilon$. In order to have cleaner formulae in the following we fix $Z(t) = t$, then the minimization in T leads to a solution of the form, using $a\,t \gg \lambda$,

$$\langle T \rangle \approx \left\{ \frac{27(3+2n)W_o e^{at}}{8(4n-3)a^2 A} \left(1 - \sqrt{1 - \frac{4(4n-3)\hat\xi}{(3+2n)^2 t^{3/2}\lambda}} \right) \right\}^{\frac{1}{3-n}} \approx \left(\frac{27W_o e^{at}}{4(3+2n)\lambda a^2 A t^{3/2}} \right)^{\frac{1}{3-n}} .$$

(A.19)

Plugging this solution into the e.o.m. for the small modulus lead by the derivatives of the exponential factors one gets the following real solution

$$\langle t \rangle \approx \left(\frac{a\hat\xi}{(3+2n)\lambda} \right)^{3/2} ,$$

(A.20)

a very similar result to the one of the standard LVS.

At the true minimun the D-term actually is not zero as the F-term part also contributes to the e.o.m for ϕ. The estimation for its correct VEV is done by refining the e.o.m. for ϕ including the leading contribution from the F-term potential, analogous to eq.(A.9). This

shows that at the vacuum $G_X \sim \epsilon^2$ and so it is D^X.

Like in the standard scenario the vacuum, so far, turns out to be an AdS one, again of $\mathcal{O}(\epsilon^3)$, since the negative term in (A.18) leads in a $at \gg 1$ expansion, and the D-term dynamics, although positive definite cannot do the job since scales like $\mathcal{O}(\epsilon^4)$.

When a non-vanishing value for the mass parameter is turned on not only a non-vanishing VEV for ρ is generated but also a term of the form,

$$V_F \supset \frac{1}{4Z(t)T^{3-n}} m^2 |\phi|^4 \,, \tag{A.21}$$

coming from $G^{\rho\bar{\rho}} |G_\rho|^2$, which being positive definite can potentially uplift the vacuum this as far there is tuning in the mass parameter of order $m \sim \epsilon^{n-1/2}$. Notice that the term (A.15) might work also since now the VEV of ρ is non zero. However, the minimization of ρ, driven by (A.15) and the linear term leads to $\rho \sim \epsilon^{5/6-n/3}$, so that (A.15), and the linear term also, scale like $\epsilon^{10/3}$ not enough for the uplift.

Unfortunately by numerical proof we found that the perturbation on the vacuum once the mass term is turned on does not allow to uplift and the minimum continue to be an AdS. This can be understood from the fact that the uplifting term goes like $V_{uplift} \sim \frac{1}{a^2 Z^3} \epsilon^3$ which cannot compete with the negative term in the moduli potential scaling as $\sqrt{t} a^2 Z^2 \epsilon^3$, and if one tries to increase the mass parameter in order to compensate this the moduli vacuum starts to be perturbed by sending their VEV to larger values so that the net effect is lost. We have check numerically, in fact, that the vacuum with a non-vanishing m, but still small, ends deeper than in the $m = 0$ case, then when increasing m only a virtual uplift is realized by the shrink of the potential due to a larger T VEV.

The D-term dynamics drives the leading contribution to the ϕ real part mass, $m_{Re(\phi)}^2 \sim (\partial_\phi G_X)^2 \sim \phi^2 / T^{2n} \sim \epsilon^{1+2n/3}$, so canonically normalized $m_\phi \sim \sqrt{\epsilon}$, like the gauge symmetry breaking scale. More precisely this real field has a small component from the real part of t, and the imaginary counterpart is the would-be Goldstone field eaten up by the now massive vector field. The other combination of ϕ and t gets a mass like for the small modulus in the standard stabilization. For the large modulus we have the same scaling for the mass as in the standard scenario. For ρ we read the mass from the quadratic term eq.(A.17), which scales like $m_\rho^2 = \epsilon^{2+2n/3}$, so normalized $m_\rho \sim \epsilon$ for both real and imaginary components.

$N_C < N_F < 3/2 N_C$ case

An interesting possibility was studied in [29], where standing in the Seiberg duality window $N_C < N_F < 3/2 N_C$ of the non-Abelian sector found vacua with exponentielly sized volumina. The dual model is described by a meson like field, ϕ, and two chiral fields, q and p, with a superpotential depending on the non-perturbative scale, Λ, a possible mass term for ϕ and a further scale μ dictated by the duality relations. As usual the non-perturbative scale has an exponential dependency on the small modulus. Then replacing the mass parameter by a dynamical field ρ, we have the following superpotential [29],

$$W = W_o + A e^{-at} \left(\frac{q\phi p}{\mu} + \rho\phi \right) \,, \tag{A.22}$$

which has an $U(1)$ symmetry characterized be the charges,

	q	p	ρ	ϕ	$e^{a\,t}$
$U(1)$	1	1	2	-1	1

so that t gets a non-linear realization identifying $\delta^t = 2/a$ as the GS coefficient for t. With this superpotential and particular modular weights, $K \supset \frac{t^n}{T_r}|Q|^2$, Krippendorf and Quevedo found vacua with exponentially large volumina and vanishing cosmological constant [29]. In the following we will give a generic study of the scalings around the vacuum. To simplify the analysis we neglect the fields q and p, as their VEV turn out to vanish [29]. Taking for $K \supset \frac{Z(t_r)}{T_r^n}|Q|^2$, and $f_X = t$ the leading contributions to the D-term potential reads

$$V_D \sim \frac{1}{t_r}\left(\frac{3}{2a}\frac{\sqrt{t_r}}{T_r^{3/2}} + \frac{Z(t_r)}{T_r^n}\big(2|\rho|^2 - |\phi|^2\big)\right)^2,\qquad\text{(A.23)}$$

neglecting also terms subleading in $1/at$. Like in the previous two cases the minimization of the potential would lead to a leading cancellation of the D-term potential due a non-vanishing ϕ VEV, then $|\phi|^2 \approx \frac{3\sqrt{t_r}}{2aZ(t_r)T_r^{3/2-n}} \sim \epsilon^{1-2n/3}\phi_o(t_r)^2$ taking *a priori* $\langle\rho\rangle \ll \langle\phi\rangle$ and $\phi_o(t_r)$ encoding $\mathcal{O}(1)$ factors. Plugging back this solution into the scalar potential one finds the following leading potential for ρ, regarding real solutions so to forget the r indices,

$$V_F \approx \frac{\phi_o^2 A^2 e^{-2a\,t}}{\lambda^2 Z(t)T^{\frac{9-4n}{2}}} + 4a\,t\frac{W_o A\,e^{-a\,t}}{\lambda^2 T^{\frac{15-2n}{4}}}\phi_o\rho + \frac{3\hat{\xi}W_o^2}{2\lambda^3 T^{9/2}}, \quad V_D \approx 4\frac{Z(t)^2}{t\,T^{2n}}\rho^4.\qquad\text{(A.24)}$$

We have tacitly neglected a term quadratic in ρ coming from $e^G G^{\phi\bar\phi}|G_\phi|^2 \supset T^{n-3}e^{-a\,t}\rho^2$ as can be check at the end, using the resulting scalings, it is indeed irrelevant. Then the minimization for ρ leads to

$$\rho = -\frac{1}{(2\lambda)^{2/3}}\left(\frac{3\,a}{2}\right)^{1/6}\frac{t^{3/4}}{Z(t)^{5/6}}\frac{\big(-W_o A e^{-a\,t}\big)^{1/3}}{T^{5(3-2n)/12}},\qquad\text{(A.25)}$$

so around this solution there is a scalar potential for the moduli from which their stabilization can be studied,

$$V_{mod} \approx \frac{3}{2}\frac{A^2 e^{-2a\,t}\sqrt{t}}{a\,\lambda^2\,Z(t)^2 T^{9/2-2n}} - \frac{3}{2\lambda^{8/3}}\left(\frac{9}{2}\right)^{1/3}\frac{(a\,W_o^2)^{2/3}t^2}{Z(t)^{4/3}}\frac{A^{4/3}e^{-4at/3}}{T^{5-4n/3}} + \frac{3W_o^2\hat{\xi}}{2\lambda^3 T^{9/2}}.\qquad\text{(A.26)}$$

As was noticed in [29] the structure of the resulting potential is quite similar to the one of the standard scenario though the powers of T do not allow for a precise analytical study. They find, however, numerically vacua with exponentially large volume which furthermore realize vanishing cosmological constant.

Let us push a bit further the analysis for the stabilization, by writing the potential for the moduli as $V_{mod} = \big(\tilde A e^{-2a\,t}T^{2n} - \tilde B e^{-4a\,t/3}T^{4n/3-1/2} + \tilde C\big)/T^{9/2}$. By requiring a vanishing cosmological constant the e.o.m. for T reads,[14]

$$\partial_T V_{mod} \approx \Big(2n\tilde A e^{-2a\,t}T^{2n} - (4n/3 - 1/2)\tilde B e^{-4a\,t/3}T^{4n/3-1/2}\Big)/T^{11/2} = 0,\qquad\text{(A.27)}$$

---

[14]The same conclusions are reached using the leading e.o.m. for t, regarding all three factors in the potential to be of the same order.

so we get that $e^{2a\,t/3} \sim T^{2n/3+1/2}$, telling us that the scaling of the two first terms in the moduli potential is the same, more precisely $\mathcal{O}(T^{-14/3}) = \mathcal{O}(\epsilon^{28/9})$. But if the cosmological constant is to be zero the third term cannot have a larger scaling so one needs to require a tuning in the parameters, for example requiring $\tilde{C} \sim T^{-1/6} \sim \epsilon^{1/9}$. Such requirement is in fact found in [29] by doing an approximate analytic study and looking for conditions for the solution to exist. In their approach they consider the possibility of changing a bit the powers of T appearing in the potential, to be precise by $T^{1/6}$, something valid as far the volume is not huge. In this regime, then, we can neglect a possible tuning in $\tilde{C}$ or in any other parameter appearing in the potential and regard them as $\mathcal{O}(1)$.

Using the full e.o.m. for ϕ we estimate the VEV for the D-term finding like in the previous cases $D^X \sim G_X \sim \epsilon^2$. Unfortunately this rough analytic approach is not fine enough so to provide a good starting point to look for numerical solutions, then we have been not able to check these properties numerically.

B Numerical tests

This section shows numerical tests for the decoupling in the models discussed analytically in the main text but for the Krippendorf-Quevedo model [29] (see section (A.2)), for which unfortunately so far we have not managed to find numerically such vacua. Although the form of the chosen Kähler potential and superpotential on each case are string inspired, we sacrifice a precise justification of the numerical parameters in favor of avoiding extra factors that can make less clear the analysis of the results.

H-sector Toy-model

In order to implement a numerical test we should choose a H sector, which better to be as realistic as possible. Our main assumption in the analysis was that the conditions $\partial_H W_{SC} + W_{SC} \partial_H K_{SC} = 0$ do not leave flat directions in the H fields. From the way the Dilaton enters in the flux induced superpotential of type-IIB superstrings [40], we need at least one more dynamical field in order to satisfy such requirement.[15] In the following examples we will take the minimal set of one Complex structure field plus the Dilaton described by the Kähler potential $\mathcal{K}_{cs} = -\log(S + \bar{S}) - \log(U + \bar{U})$. The superpotential, inspired from explicit calculations on flux compactification [52], is given by

$$W_{cs}(U,S) = a_{cs}U^2 + b_{cs}U + S\big(f_{cs}U^2 + d_{cs}U + e_{cs}\big). \tag{B.1}$$

With numerical parameters that in all following examples we will take as $a_{cs} = 1$, $b_{cs} = -\frac{1}{2}$, $d_{cs} = -3$, $f_{cs} = 1$ and $e_{cs} = 5/2$, so that lead to the fixed values, solutions to the F-flatness conditions, $S_o = U_o = 1$, and more over $W_{cs}(U_o, S_o) = W_o = 1$.

The other common feature in the following examples is the matter independent Kähler potential, given by the one of type-IIB compactifications with orientifolds and the leading α' corrections, eq.(2.15), with the CY volume (2.16). Notice that all these features are fairly natural and generic, and no particular intention drives its election.

---

[15]Fixing S alone is still possible if one allows a constant part in the superpotential, something that obviously we want to avoid.

	$\langle X \rangle$	$\Delta \langle X \rangle$	F_c^X	ΔF_c^X
U	$1 - 4 \cdot 10^{-20}$	$-4 \cdot 10^{-20}$	$1 \cdot 10^{-41}$	$-$
S	$1 - 1 \cdot 10^{-19}$	$-1 \cdot 10^{-19}$	$3 \cdot 10^{-41}$	$-$
T	$7.5 \cdot 10^{13}$	$2 \cdot 10^{-18}$	$1 \cdot 10^{-21}$	$3 \cdot 10^{-18}$
t	3.21	$7 \cdot 10^{-20}$	$1 \cdot 10^{-33}$	$5 \cdot 10^{-18}$

Table 1. VEV's of the fields and canonical normalized F-term, and their relative shifts compared to the simplified model. The Q sector is not reported given the triviality of the results. Here and in the main text $\Delta X \equiv (X - X_{sim})/X$. All quantities are in natural units.

B.1 Charged large modulus

For these models we will take the following particular choice,

$$ K = K_{mod} + \frac{1}{(U + \bar{U})^\eta \mathcal{V}^n} |Q|^2 \,, \quad W = W_{cs} - A U e^{-a t} \,, \tag{B.2} $$

where we have introduced the mixing between the complex structure moduli and the matter fields in the Kähler potential, and a further coupling in the superpotential with the small modulus. The Killing vectors for the systems, with ordering $\{U, S, T, t, \phi\}$, are $\{0, 0, i\delta^T/2, 0, i q_Q Q\}$. The gauge kinetic function is chosen to be $f_X = T + \kappa S$, κ a real number.

Vanishing Q VEV

If the GS coefficient is taken as positive then a negative charge for Q leads to the vanishing VEV case presented in appendix A.1 and whose decoupling was studied in section 3.2.1. We take the following values for the parameters, besides the ones already fixed in the W_{cs} superpotential,

$$ \lambda = 1 \,, \quad A = 1 \,, \ a = 5\pi \,, \ \xi = 2 \,, \ \kappa = 1 \,, \ \eta = 1 \,, \ n = \frac{2}{3} \,, \ q_Q = -1 \,, \ \delta^T = 1 \cdot 10^{-11} \,. \tag{B.3} $$

The value for the GS coefficient δ^T is chosen such to fine tune the cosmological constant. Table 1 shows the solution for the vacuum and the deviation from the simplified model where the S and U superfields are fixed to $So = U_o = 1$, with vanishing spinor and auxiliary components. The canonical normalized auxiliary fields VEV, $F_c^I = |G_{I\bar{I}} F^I F^{\bar{I}}|^{1/2}$ no sum, are also shown. The system corresponds to an ϵ parameter, as defined in the main text, of roughly $\mathcal{O}(10^{-21})$, expecting then corrections to the simplified model of this order. The physical mass spectrum in GeV units is

$$ m_{t_i} = 18.5 \cdot 10^{-2} \,, \qquad m_{t_r} = 18.4 \cdot 10^{-2} \,, \ m_Q = 8.8 \cdot 10^{-2} \,, \qquad m_{T_r} = 7.0 \cdot 10^{-14} \,, $$
$$ m_{\tilde{S}_r} = 3.4 \cdot 10^{-2} \,, \qquad m_{\tilde{S}_i} = 3.0 \cdot 10^{-2} \,, \ m_{\tilde{U}_i} = 4.5 \cdot 10^{-3} \,, \qquad m_{\tilde{U}_r} = 8.0 \cdot 10^{-4} \,. $$

where $\tilde{S}$ and $\tilde{U}$ are defined by a sum of the original S and U, but with main component S and U respectively, and the subscript r (i) denote the real (imaginary) field components.

The imaginary component of T is the would-be Goldstone boson so is exactly massless. The shifts compared to the simplified model are

$$\Delta m_t = 7 \cdot 10^{-18} \,, \quad \Delta m_Q = 5 \cdot 10^{-18} \,, \quad \Delta m_{T_r} = 8 \cdot 10^{-18} \,, \qquad \text{(B.4)}$$

defining $\Delta X \equiv (X - X_{sim})/X$. For the gravitino mass, the D-auxiliary field VEV and the cosmological constant we have, all in natural units,

$$
\begin{aligned}
D^X &= 1.3 \cdot 10^{-39} \,, & \Delta D^X &= 5 \cdot 10^{-18} \,, \\
m_{3/2} &= 7.7 \cdot 10^{-22} \,, & \Delta m_{3/2}^2 &= 3 \cdot 10^{-18} \,, \\
V_0 &= -5.5 \cdot 10^{-65} \,, & \Delta V_0 &= 9 \cdot 10^{-18} \,.
\end{aligned}
\qquad \text{(B.5)}
$$

Although a quite unrealistic scenario since presents a SUSY breaking scale in the $10^{-3} GeV$ region and a too light moduli, such small ϵ helps to split the scales and clearly shows the scaling of the errors in the simplified version. Any way we have check for different values of ϵ these scaling. From the results we see that the mistake are few orders of magnitude greater than the predicted values, reaching even three orders in worst cases. Still, it is clear that all errors are correlated and of the same order. In order to understand such difference with the prediction we should remember the numerical factors carefully extracted in section 2.2. There we found that the error was enhanced by a factor $\sim \frac{8}{3} \lambda t^{3/2} a^4 \tilde{A}^2$ with $\tilde{A}$ a numerical factor missed in the analytical solution for e^{at}. This factor accounts for an increasing of two orders in the result. The extra order is easily understood from the fact that the error in the small modulus propagates in the e.o.m. for the large one with a factor a. Indeed the larger error in the fields VEV corresponds to T, as can be seen in table 1, and all the following samples, and then it propagates to the auxiliary fields VEV's, masses and more drastically in the cosmological constant, being the first two suppressed by $e^{G/2} \sim 1/T^{3/2}$ and the last one by $e^G \sim 1/T^3$, so that roughly $\Delta V \sim -3V \Delta T$.

Non-vanishing Q VEV

With the opposite sign for the Q charge the VEV of Q is non- zero and an F-term uplifting is possible, section A.1. The parameters taken for this example are

$$\lambda = 1 \,, \quad A = 1 \,, \ a = 3\pi \,, \ \xi = 2 \,, \ \kappa = 1 \,, \ \eta = 1 \,, \ n = \frac{4}{9} \,, \ q_Q = 1 \,, \ \delta^T = 5 \cdot 10^{-6} \,. \quad \text{(B.6)}$$

The value for the GS coefficient δ^T again is chosen such to tune the cosmological constant. Table 2 shows the VEV's of the fields and their deviations from the simplified model. The system corresponds to an ϵ, as defined in the main text, of roughly $\mathcal{O}(10^{-12})$, expecting corrections to the simplified version of this order. The physical mass spectrum in GeV units is

$$
\begin{aligned}
m_{Q_r} &= 4.2 \cdot 10^7 \,, & m_t &= 3.6 \cdot 10^7 \,, \ m_{Q_i} = 9.8 \cdot 10^5 \,, & m_{T_r} &= 0.6 \quad , \\
m_{\tilde{U}_r} &= 1.1 \cdot 10^7 \,, & m_{\tilde{U}_i} &= 9.9 \cdot 10^6 \,, \ m_{\tilde{S}_i} = 1.5 \cdot 10^6 \,, & m_{\tilde{S}_r} &= 2.6 \cdot 10^5 \,. \quad \text{(B.7)}
\end{aligned}
$$

again $\tilde{S}$ and $\tilde{U}$ are defined by a sum of the original S and U, but with main component S and U respectively and the subscript r (i) denote the real (imaginary) field components.

	$\langle X \rangle$	$\Delta\langle X \rangle$	F_c^X	ΔF_c^X
U	$1 - 1 \cdot 10^{-11}$	$-1 \cdot 10^{-11}$	$1 \cdot 10^{-24}$	$-$
S	$1 - 5 \cdot 10^{-11}$	$-5 \cdot 10^{-11}$	$3 \cdot 10^{-24}$	$-$
T	$1.59 \cdot 10^{8}$	$-4 \cdot 10^{-10}$	$4 \cdot 10^{-13}$	$6 \cdot 10^{-10}$
t	3.21	$-2 \cdot 10^{-11}$	$1 \cdot 10^{-20}$	$8 \cdot 10^{-10}$
Q	$1.2 \cdot 10^{-4}$	$6 \cdot 10^{-11}$	$1 \cdot 10^{-20}$	$8 \cdot 10^{-10}$

Table 2. VEV's of the fields and canonical normalized F-term, and their relative shifts compared to the simplified model with $\Delta X \equiv (X - X_{sim})/X$. All quantities are in natural units.

The imaginary component of T is the would-be Goldstone boson so is exactly massless. The shifts compared to the simplified model are

$$\Delta m_t = 5 \cdot 10^{-10}\,, \quad \Delta m_{Q_r} = 4 \cdot 10^{-10}\,, \quad \Delta m_{Q_i} = 6 \cdot 10^{-10}\,, \quad \Delta m_{T_r} = 6 \cdot 10^{-10}\,. \quad \text{(B.8)}$$

For the gravitino mass, the D-auxiliary field VEV and the cosmological constant, in natural units, we have

$$
\begin{aligned}
D^X &= 2.1 \cdot 10^{-26}\,, & \Delta D^X &= 1 \cdot 10^{-9}\,, \\
m_{3/2} &= 2.5 \cdot 10^{-13}\,, & \Delta m_{3/2}^2 &= 6 \cdot 10^{-10}\,, \\
V_0 &= -4.3 \cdot 10^{-39}\,, & \Delta V_0 &= 1 \cdot 10^{-9}\,.
\end{aligned}
\quad \text{(B.9)}
$$

This, now more realistic, scenario, with a SUSY breaking scale of $\mathcal{O}(10^6)GeV$ presents exactly the same features in the errors as the previous case. This is expected as the largest corrections come from the same source independent of the matter fields and gauge dynamics. Thus, the explanation for the factors showing the errors larger than expected ones follows verbatim here.

For the record we have done numerically the very same model changing the gauge kinetic function to $f = t + \kappa S$. This in order to check the claim that is the scaling of the VEV for D^X auxiliary field what matters. Otherwise, as explained in footnote 10, with $f \sim t$ the corrections would rather scale like $\mathcal{O}(\epsilon^{2/3})$. We find in fact numerically the very same results as the ones just presented, being the only drastic change the VEV of G_X.

B.2 Charged small modulus

The system is the one described in detail in appendix (A.2), so besides checking explicitly in numbers the decoupling we take the chance to see all features found analytically there. For the matter fields the Kähler potential is taken to be

$$K_Q = \frac{t_r}{T_r} \left(|\rho|^2 + U_r |\phi|^2 \right) \,. \quad \text{(B.10)}$$

The superpotential is taken as

$$W = W_{cs}(U, S) - A S \frac{e^{-at}}{\phi^2} - m U^2 \rho \phi^2 \,, \quad \text{(B.11)}$$

	$\langle X \rangle$	$\Delta\langle X \rangle$	F_c^X	ΔF_c^X
U	$1 - 1 \cdot 10^{-5}$	$-1 \cdot 10^{-5}$	$2 \cdot 10^{-12}$	$-$
S	$1 - 4 \cdot 10^{-5}$	$-4 \cdot 10^{-5}$	$5 \cdot 10^{-12}$	$-$
T	11247	$-2 \cdot 10^{-4}$	$7 \cdot 10^{-7}$	$3 \cdot 10^{-4}$
t	2.36	$-2 \cdot 10^{-5}$	$4 \cdot 10^{-11}$	$4 \cdot 10^{-4}$
ϕ	$4.6 \cdot 10^{-2}$	$7 \cdot 10^{-5}$	$9 \cdot 10^{-11}$	$4 \cdot 10^{-4}$
ρ	$7.1 \cdot 10^{-3}$	$8 \cdot 10^{-5}$	$2 \cdot 10^{-10}$	$3 \cdot 10^{-4}$

Table 3. VEV's of the fields and canonical normalized F-terms, and their relative shifts, derived by a numerical analysis with $\Delta X \equiv (X - X_{sim})/X$. All quantities are in natural units.

and the gauge kinetic function $f_X = t + \kappa S$, which is different from the one taken in (A.2) but its t dependency and scaling in ϵ are the same though. The parameters taken are

$$\lambda = 1, \quad A = 1, \quad a = 3\pi, \quad \xi = \frac{6}{5}, \quad \kappa = 1, \quad m = 268 \cdot 10^{-5}, \quad \delta^t = \frac{2}{3\pi}, q_\phi = -\frac{1}{2}, q_\rho = 1. \tag{B.12}$$

We take a value for m that in principle would work to uplift the vacuum, being $\mathcal{O}(\epsilon^{1/2})$. However the relative uplift to the zero m vacuum turn to be only a virtual one being only due to the shift caused on the VEV of T which moves to larger values. In fact, as advertised before, for m slightly smaller than $\epsilon^{1/2}$, such that its perturbation on T is less relevant the vacuum goes slightly deeper than in the zero m case, the same happening with positive or negative values for m. As defined in appendix (A.2) the GS coefficient is related to the parameter a by $\delta^t = 2/a$, and the ϕ and ρ charges are fixed as well, but we report them again for completeness.

Table 3 shows VEV's of the fields and the deviation from the simplified model. The canonical normalized auxiliary fields VEV are also shown. The system corresponds to an ϵ, as defined in the main text, of roughly $\mathcal{O}(10^{-6})$, to be precise $8 \cdot 10^{-7}$, expecting corrections to the simplified model of order $\mathcal{O}(\epsilon^{2/3}) \sim 10^{-5}$. We check then that indeed the field ϕ is stabilized mainly by the nearly vanishing D-term condition, $\phi \approx (a^2 t_r T_r)^{-1/4}$. For the rest of the quantities the scalings are also in agreement with the one found in appendix (A.2). The physical mass spectrum in GeV is

$$m_{\tilde{\phi}_r} = 6.3 \cdot 10^{14}, \qquad m_{\tilde{t}} = 4.6 \cdot 10^{13}, \quad m_{\rho_i} = 2.2 \cdot 10^{13}, \qquad m_{\rho_r} = 2.0 \cdot 10^{12},$$
$$m_{\tilde{U}_r} = 1.9 \cdot 10^{13}, \quad m_{\tilde{U}_i} = 1.7 \cdot 10^{13}, \quad m_{\tilde{S}_i} = 2.5 \cdot 10^{12}, \qquad m_{\tilde{S}_r} = 4.4 \cdot 10^{11}, \quad \text{(B.13)}$$
$$m_{T_r} = 1.2 \cdot 10^9.$$

where $\tilde{\phi}$ ($\tilde{t}$) is a linear combination of ϕ and t with ϕ (t) as main component, and $\tilde{S}$ and $\tilde{U}$ are defined by a sum of the original S and U, but with main component S and U respectively. The subscript r (i) denote real (imaginary) field components. The imaginary component of $\tilde{\phi}$ is the would-be Goldstone boson so is exactly massless, as well the imaginary part of T.

The shifts compared to the simplified model are

$$\Delta m_{\tilde{\phi}_r} = 2 \cdot 10^{-4}, \quad \Delta m_{\tilde{t}} = 3 \cdot 10^{-4}, \quad \Delta m_\rho = 2 \cdot 10^{-4}, \quad \Delta m_{T_r} = 3 \cdot 10^{-4}. \tag{B.14}$$

For the gravitino mass, the D-auxiliary field VEV and the cosmological constant we have, in natural units,

$$D^X = 4.5 \cdot 10^{-13}\,, \qquad\qquad \Delta D^X = 5 \cdot 10^{-4}\,,$$
$$m_{3/2} = 4.2 \cdot 10^{-7}\,, \qquad\qquad \Delta m_{3/2}^2 = 3 \cdot 10^{-4}\,, \qquad\qquad \text{(B.15)}$$
$$V_0 = -1.7 \cdot 10^{-20}\,, \qquad\qquad \Delta V_0 = 7 \cdot 10^{-4}\,.$$

This scenario with high SUSY breaking scale, $\mathcal{O}(10^{12})GeV$, shows an interesting situation that might happen with the corrections. Although the corrections fall in predicted order, i.e., $\mathcal{O}(\epsilon^{2/3})$, the main corrections come from the $\mathcal{O}(\epsilon)$ contribution coming again mainly from the $G_{t\bar{t}}|F^t|^2$ terms. This is check by performing the same numerical search but setting $m = 0$. Analytically it can be understood by looking to enhancement factors as we did before to explain the numerical results for the previous two cases. For the corrections coming from the mass parameter we have $\Delta \sim \frac{a^2\sqrt{t}\tilde{A}\tilde{m}}{\tilde{\rho}}\epsilon^{2/3}$, with $\tilde{A} \equiv e^{-a\,t}/\epsilon^{4/3}$, $\tilde{m} \equiv m/\epsilon^{1/2}$, $\tilde{\rho} \equiv \rho/\epsilon^{1/2}$, and for the present example the correction factor is $\sim 1/2$. On the other hand, the ones from $G_{it} \sim ae^{-at}/\phi^2 + \epsilon^{2/3}\phi^2$ leads to corrections going like $\Delta \sim a^4 t^{3/2}\tilde{A}^2\epsilon$, which enhance them by almost two orders. The result is that both errors enter in the same order though the ones scaling as $\mathcal{O}(\epsilon)$ turn out to be slightly bigger.

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